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**Do Marital Prospects Dissuade Unmarried  
Fertility?**

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## **ABSTRACT**

Unmarried fertility was a lot lower in the 1970s than in the 1990s. It was also the case that unmarried mothers had much lower marriage rates than non-mothers, a differential that has largely vanished over time. Could this marriage-market penalty have been strong enough to explain why unmarried fertility rates were lower then? To explore this issue, we introduce a new model of fertility and marriage, based on directed search. Relative to the existing literature, the essential contributions of the model are to allow for accumulation of children over the lifecycle and for the marriage of single mothers. We use the model, in conjunction with US survey data, to explore the impact of marital prospects on the fertility decisions of unmarried women. We find that the decline, from the 1970s to 1995, in marriage rates of unmarried women with no children, can account for the dramatic rise in unmarried women's share of births over that period.

## **KEYWORDS**

Two-Sided Search; Divorce; Family; Family Economics; Household Formation; Marital; Marital Dissolution; Marriage; Marriage Rate; Marriage and Technological Progress; Maternity; Motherhood; Mothers; Parent; Premarital; Single Mother; Single Parent; Fertility; Family Planning; Household Production; Intra-household Allocation.

## **EDITORIAL NOTE**

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# DO MARITAL PROSPECTS DISSUADE UNMARRIED FERTILITY?

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# 1 Introduction

"But step-families are different. The stepparent has shopped for a spouse, not a child; the child is a cost that comes as part of the deal...step-parenthood is the strongest risk factor for child abuse ever identified".

–Steven Pinker, *How the Mind Works*, 1997

According to the US census, about 50% of marriages now involve a spouse who has been previously married, and 30% of U.S. children are not living with both parents. These two facts represent a rapid shift of actual demographic behavior away from the standard represented in economic marriage models, where marriage occurs once and parenthood, if included at all, is restricted to married couples.

While fertility decline and rising divorce rates among married couples are among the behavioral shifts driving these trends, shifts in the behavior of singles, such as declining marriage rates and rising fertility rates, appear to be more important, according to Ventura and Bachrach (2000). It seems plausible therefore that at least part of this transformation is driven by equilibrium considerations; marital prospects are likely to play a strong role in the fertility decisions of single women, and behavior when married is likely to be increasingly influenced, as divorce and re-marriage become more frequent, by the prospect of a return to the marriage market.

In this paper we provide a simple model of the role of these marital prospects on both married and single fertility. The mechanism underlying our marriage market model is directed search; men decide which type of women to court, where by "type" we mean the number of children a woman already has. Single men prefer women without children, but competition for wives reduces both the probability of marriage and the husband's share of the surplus, so some may choose to court single mothers. Women without children therefore attract more suitors, marry at a higher rate, and get more surplus within marriage. They face therefore a disincentive for unmarried fertility whose strength is increasing in the gains from marriage.<sup>1</sup>

Relative to the previous marriage literature, the main contribution is that our model can allow for arbitrarily high numbers of children and transitions in and out of marriage. This means it is relatively straightforward to match our model to annual data and thus measure the importance of these interactions.

We provide an application of our model to demonstrate this feature; a numerical analysis of the change in marital regime between 1973 and 1995. In our empirical section, we analyze the 1973 and

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<sup>1</sup>This is consistent with the empirical results of Rosenzweig (1999), which finds that a fall in marital prospects significantly raises the chances that young US women will choose non-marital fertility.

1995 waves of the National Survey of Family Growth. By estimating probit regressions for marriage and giving birth, we are able to show that neither the decline of marriage rates nor the rise in unmarried birth rates can be attributed to shifting education patterns, age distribution or tendency to cohabitation. We use the estimates from 1973 to calibrate the model; the calibration exploits the difference in marriage rates between mothers and non-mothers to identify the importance of marital prospects in marriage. The results show that the model can generate a close match, not only to the statistical targets, but also to some important non-targeted moments.

The model's free parameters include preference shocks that shift the average marriage rate, as well as the marriage penalty for single mothers, without affecting the directly affecting the payoffs for fertility. We can therefore isolate the effect on fertility of the decline from the 1970s to the 1990s in the marriage rates for non-mothers by re-setting these parameters to generate the marriage and divorce rates of the 1990s. In the benchmark model, this change results in a significant increase in birth rates to unmarried women, and can explain virtually all of the increase in the unmarried share of total births.

Of course this exercise can not explain why the change in marriage patterns might have occurred, only that they are sufficient to explain the rise of unmarried fertility. A related paper in progress, Kennes and Knowles (2011) extends the model presented here to consider the search for sexual partners, and asks whether improved contraception for single women can explain the transition in marital patterns. In the current model, by contrast, improved birth-control generates very little change in marital patterns, and the unmarried birth rates fall rather than rise.

While there is a large literature on the determinants of unmarried parenthood, very few published papers consider the impact on marriage-market equilibria posed by the choice between fertility inside and outside of marriage. Most papers that consider fertility in the context of marriage-market equilibria assume fertility is either exogenous, as in Weiss and Willis (1997) or occurs within marriage only, or both, as in Aiyagari, Greenwood, and Guner (2000) and Chiappori and Weiss (2006). In that sense this paper is more closely related to Akerlof, Yellen, and Katz (1996) and Chiappori and Oreffice (2008) which model pre-marital fertility. Greenwood and Guner (2005) models the segregation of young singles into sexually promiscuous and abstinent groups in response to improvements in contraception technology. All of the latter papers however abstract from married fertility decisions.

Among the few equilibrium models that consider both margins of fertility, there is Neal (2004), which examines the interaction between welfare payments and marriage-market equilibria that differ in unmarried fertility rates, Greenwood, Guner, and Knowles (2000) (GGK hereafter), which shows how marriage-market dynamics and human-capital investment perpetuate the effect of rising welfare payments on unmarried fertility.<sup>2</sup> An important feature of these papers is that they model, inter

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<sup>2</sup>The theoretical framework underlying Greenwood, Guner, and Knowles (2000) is developed in Greenwood, Guner,

alia, the impact of pre-marital fertility on the household allocations of married couples through the mechanism of marriage-market equilibrium. However these papers suffer however from an extreme compression of the lifecycle; only GGK allows for divorce and remarriage, but even there, marriage is only allowed in two periods and divorce only in one.

The exception to the rule is Regalia and Ríos-Rull (1999) , which develops a life-cycle model to analyze the impact of wage inequality on marriage, divorce and fertility. That paper does allow for both fertility margins, keeps track of previous children, and allows for a random-search style of marriage equilibrium. However the focus there is on wage inequality and heterogeneity, which is outside the scope of the current paper.

In the search-and-matching literature, models with repeated matching opportunities are entirely standard, however this is typically achieved by abstracting from choices, such as investment, that permanently change the state of an agent. In the marriage-market model of Chade and Ventura (2005), for instance, based on the job-search framework of Burdett and Wright (1998), agents experience an infinite succession of marriages and divorces in response to changes in match quality, which is represented by an iid random variable. The matching literature has also considered the analysis of marriage markets with ex-ante heterogeneous agents, as in Burdett and Coles (1997), where agents sort into marriages on the basis of quality differences which are assumed to be permanent. Recently the literature has begun to consider matching with pre-marital investments, as in Burdett and Coles (2001), but that literature does not consider the margin between investments inside and outside the match, which is the mathematical analog of the fertility decision in the current paper.<sup>3</sup>

## 2 Empirical Background

The decline in marriage rates is well-documented phenomenon; for a cohort analysis, see Schoen and Standish (2001), who show that female marriage rates declined from about 25% per year for women aged 20-25 in the 1946-50 birth cohort, to 10% per year for the corresponding women in the 1969-73 birth cohort. In figure 1 we show a similar result for age-marriage profiles at annual frequencies, based on the U.S. Census; the marriage rate at age 25 declines from above 20% to about 8% from oldest to youngest cohort.

Casual empirical evidence for the basic mechanism in the model comes from the lower marriage rates and Knowles (2003). Two closely-related papers that use a similar framework are Caucutt, Guner, and Knowles (2002) and Guner and Knowles (2008).

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<sup>3</sup>A related literature on matching with pre-marital investments, such as Iyigun and Walsh (2007), does not allow for the investment to take place after marriage, and so cannot account for variation over time in the timing of investments like fertility.

of single mothers, the lower share of the marriage output allocated to single mothers when they do marry and a host of anecdotes across many cultures illustrating the tension between children and step parents (see Pinker (1999) for a summary)<sup>4</sup>. Beaujouan (2009) finds that re-partnering rates in France are significantly lower for single mothers than for child-less women or single men with or without children. She shows however that this asymmetry between men and women is entirely accounted for by the fact that single mothers are much more likely to live with the children than are single males. Similarly Browning and Bonke (2006) find that having children from a previous marriage does not reduce the intra-household allocation to Danish husbands in subsequent marriages, but has a strong negative effect on the allocation for wives. Again the explanation appears likely to be co-residence of mothers with their children, although the survey lacks the variables required to test this hypothesis.

Despite such disincentives, the fraction of U.S. births accounted for by unmarried women has risen steadily, from 5% in 1945 to 40% in 2009, according to Ventura (2009). They show that there has been steady growth in the unmarried birth rate since 1940, and in the unmarried share of births since at least 1960. This latter increase has proceeded at roughly the same rate in all age groups from 15-17 to 30-34, though the timing of the change is later in the older groups.

Since 1980, cohabitation has played an increasingly important role, but in 1990-95, according to Ventura (2009), only 39% of unmarried births were to cohabiting women, so the majority were to women who were neither married nor cohabiting. The fraction of children who live with cohabiting parents remains extremely low; even in 2010, according to the Current Population Survey, unmarried parents with children constituted only 2.1% of households.<sup>5</sup>

Of course the population of unmarried women has changed in other ways over time, with the rise of women's college education and the aging of the baby boom, as well as divorce and cohabitation. To isolate the changes in marriage and fertility rates from the composition effects, it is necessary to use regression methods on micro data. In this section we compare the behavior of unmarried women in two waves of the National Survey of Family Growth, 1973 and 1995, which have complete fertility and marital histories for samples of women aged 15-44 at the time of the survey.

While the 1995 wave was designed to be representative of the US population as a whole, the 1973 wave excluded never-married women with no cohabiting own children<sup>6</sup>. We reweight this survey by (1)

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<sup>4</sup>Pinker: In one study of emotionally healthy middle-class families in the U.S., only half of the stepfathers and a quarter of the stepmothers claimed to have "parental feelings" toward their step-children.

<sup>5</sup>Source: U.S. Census Bureau, Current Population Survey, 2010 Annual Social and Economic Supplement, Table FG10.

<sup>6</sup>The sample design for 1973 was described as follows: 9,797 women aged 15-44 living in the coterminous United States who were either currently married, previously married, or never married but had offspring living in the household in 1973.



using the 1970 Census to create dummy observations for these excluded women by age and education level, so that the proportions of these women in each cell match the 1970 census, and (2) reweighting the survey observations to account for the women excluded from the population in the denominator. Figure 2 shows that this procedure results in a very close match between the Census and the reweighted NSFG sample for all the age-education groups. Deviations are most noticeable for the smallest cells, where the fraction of unmarried women with children in the original sample is most vulnerable to sampling error.

In Figure 3 we show that whether we include cohabitation as singles or not has little impact on the average behavior of singles; in the top panel, marriage rates are plotted against the number of children. The bottom panel shows that births per year is much higher for cohabitants, but because of their small number in 1995, relative to the large mass of singles who did not have children, they do little to shift the plots of singles. Of course the same argument applies to 1973, when the cohabitation rate was much lower.

We estimate probit regressions by month for the birth of a child, and for marriage, for 1970-73 and 1990-95 (these date ranges correspond to periods for which the surveys collected extra variables). Earlier surveys did not interview single women, and so are not useful for this purpose. The regression equations include dummy variables to control for whether the woman is cohabiting, whether she was previously married and whether she graduated from high school, attended college or attained a bachelor's degree.<sup>7</sup>

The resulting estimates are presented in Table 3. In the 1970s, having a child has a strong positive association with the probability of a future childbirth (estimate=0.758), and a strong negative association (-0.41) with future marriage. To put this in perspective, the effect of college graduation is quite small by comparison: (-0.39 for births and -0.27 for marriages). The effects of cohabitation are large (1.1 for births and -2.96 for marriages), but the rate of cohabitation is negligible in the 1973 survey. The marginal effects of additional children tend to be significantly smaller, except for 5 or more kids, where sampling error and selection are likely to confound the interpretation of the estimates.

In 1995, the effect of children on future births has shrunk to a third of its former size, and that on marriages has practically vanished to zero. The cohabitation effects are also tiny by comparison with the 1970s. While the impact of a college degree on births is still strongly negative, the effect on marriage has also faded to zero.

In Figure 4 we show projected age profiles using the estimated coefficients. The figure compares

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<sup>7</sup>The 1973 wave does not record whether the respondent is attending school, nor her eventual attainment. Instead we know her years of schooling completed. We assume she is not attending if her age exceeds years of schooling by 6 years or more, while we use thresholds of 12 and 16 years as proxies for high-school graduation and attainment of a bachelor's degree, respectively.

women with no children to those with one child already. In both cases the comparison is for women with a diploma but no college attendance. It is apparent from panel (a) that marriage rates for women without children fell considerably; the marriage hazard rate at age 22 declined from 30% per year to about 12%, while panel (b) shows virtually no change for single mothers. With regards to birth rates, those of childless women aged 22 quadrupled, from 2% to 8% annually, while for single mothers, the birth rates fell significantly, from 18% for 22 years old, to 13%.

These figures indicate that neither trends in education nor in cohabitation can explain away the dramatic rise in unmarried fertility since the 1970s; the change in behavior within the education/age/cohabitation cells is an important component of the overall change. Analogous results for married women's fertility are reported in the appendix (Tables A2 and A3). Those tables show that fertility profiles for married women, conditional on zero or one child, have been relatively stable.

### 3 A Life-Cycle Model of Fertility and Marriage

The population of agents consists of infinitely-lived adults, with a continuum of each sex denoted by  $\{M, F\}$  and mass  $N_M$  and  $N_F$ . Life is divided into discrete periods. Women are of sex  $f$  and may produce up to  $K$  children.

There are three types of households; single males, single females, and married couples. Married adults live together as husband and wife with all the children ever born to the female spouse.

Each period, couples experience random shocks to the quality  $\tilde{q}$  of the marriage, which consist of utility flows to each spouse. Households exit permanently from active status, i.e. "become sterile" with probability  $\delta$  each period; we assume they then enjoy their current utility flow forever. They are replaced by an equal inflow of unmatched men and child-less women.

Let  $k$  be the number of kids in a married-couple household, and  $k_m \leq k$  be the number of the husband's biological (own) kids. Children are born to women at rate  $\pi^F$  each period. The cost of reducing the fertility rate  $\pi^F$  below a baseline fertility rate  $\widehat{\pi}_k^{mF}$  is given by the function  $\Theta(\pi^F | \widehat{\pi}_k^{mF})$ , where  $m$  indicates the marital status and  $k$  the parity (number of children to date) of the woman.

#### 3.1 Preferences

The indirect utility functions  $\tilde{u}_{SM}$ ,  $\tilde{u}_{SF}(k)$  and  $\tilde{u}_M(k, k_m)$ , for, respectively, single males, single women and married-couples, to represent the maximized utility flow each period from consumption and children. The critical assumption is that children generate more utility within a marriage than without:

$$\tilde{u}_{SF}(k+1) - \tilde{u}_{SF}(k) < \tilde{u}_M(k+1, k_m-1) - \tilde{u}_M(k, k_m)$$

What we have in mind here is the idea that parents get less utility from step children than from their own children, so that an additional child within a marriage raises the father’s utility more than a pre-existing child would. We also follow Aiyagari, Greenwood, and Guner (2000) and many other papers in assuming that children outside the household do not enter the parent’s utility function.

Utility within married couples is perfectly transferable. This means that utility of the couples can be traded off on a one-for-one basis. Assuming full commitment, therefore, all allocations of the surplus can be efficiently achieved by maximizing the equally-weighted sum of the welfare of husband and wife.

The stochastic process for match quality  $\tilde{q}$  is assumed to contain both a persistent component  $q \in Q \equiv \{q_1, q_2, \dots, q_{N_q}\}$  and an iid component  $\varepsilon \in R$ . For new marriages, the process for  $q$  is indexed by  $\bar{q} \in Q$ .

Fertility decisions are made after the match quality has been realized. Computational considerations restrict the persistent component to a discrete support, so the iid component of the process is a computational convenience that allows for a continuous support for match quality, which is critical for comparative statics.

### 3.2 Frictional assignment

Transitions between household types occur through marriage and divorce. Marriage occurs through a directed-search matching mechanism based on Julien, Kennes, and King (2000). We refer to the pool of all single males and females as the "marriage market" ; this consists of new entrants and those who were single or became divorced last period and remain active this period. The number of single-female households with  $k$  children is denoted by  $N_F(k)$ . We also assume that while entry into the marriage market is costless for women, there is an entry cost  $\gamma > 0$  that single men must pay.

Each period there is random matching within  $K + 1$  marriage sub-markets, one for each type of single woman. For each sub-market  $k$ , the number of women is denoted  $N_F(k)$ . Suppose there are  $N_M(k)$  men who enter marriage markets of type  $k$ . Let the ratio of men to women in sub-market  $k$ , ie the “queue length” for market  $k$  be denoted  $\phi_k \equiv N_M(k)/N_F(k)$ .

At the beginning of the period, each single man (“suitor”) is randomly assigned to a woman within the market he has chosen. Each single woman therefore starts the period with  $k$  children and an integer number of suitors  $z$ . The probability distribution over  $z$  is given by  $\omega_z(k)$ , and women auction the match to the highest bidder. The probability that the husband receives the surplus is therefore  $\omega_0(k)$ , the probability that he was the only entrant, while the probability that the woman matches is given by  $1 - \omega_0(k)$ , the probability that she has at least one suitor. A man will match with probability  $1/z$ , which on average is equal to  $(1 - e^{-\phi_k})/\phi_k$ . A newly matched couple learns the first realization of their

marriage quality  $(q, \varepsilon)$ , at which point they decide whether to marry. If they do not marry, they split up and spend the remainder of the period as single households.

### 3.3 Expected payoffs

Since utility is transferable, decisions within the marriage, such as fertility and divorce, maximize the expected surplus, contingent on  $(k, k_m)$  and the current values of the match-quality variables,  $(q, \varepsilon)$ .

It is convenient to divide each period into the stage before and the stage after marital events. Let the values on entering the period, for men and women, respectively, be denoted  $V_{SM}^E(k, \phi)$  and  $V_{SF}^E(k, \phi)$ , where  $k$  denotes the sub-market the single person is participating in. Let  $Y^E(k, k_m|q_{-1})$ , denote the expected value, on entering the period, of a marriage consisting of a woman with  $k$  kids of her own, of which  $k_m$  are children of her current husband, conditional on  $q_{-1}$ , the previous-period's realization of  $q$ .

Let the effective discount rate be denoted  $\tilde{\beta} \equiv \beta(1 - \delta)$ . Finally, let  $EU(\pi^F, k, k_M)$  represent the expected utility in each period, net of the quality flow and conditional on fertility choice  $\pi^F$ :

$$EU(\pi^F, k, k_M) \equiv \pi^F u_M(k+1, k_M+1) + (1 - \pi^F) u_M(k, k_M) - \Theta(\pi^F)$$

Let  $Y^R(k, k_m|q, \varepsilon)$  be the value of the marriage, given optimal fertility decisions, after the match quality shocks  $(q, \varepsilon)$  are realized but before the fertility realizations. We can write this in terms of the flows we have just defined as:

$$\begin{aligned} Y^R(k, k_m|q, \varepsilon) = & \max_{\pi^F} \{ EU(\pi^F, k, k_M) + q + \varepsilon \\ & + \tilde{\beta} [\pi^F Y^E(k+1, k_m+1|q) + (1 - \pi^F) Y^E(k+1, k_m+1|q)] \} \end{aligned} \quad (1)$$

. The alternative to any given marriage is to remain single for the period. Let  $V_{SM}^R(\phi)$  and  $V_{SF}^R(k, \phi)$  denote the continuation values as singles for men and women, respectively, at the close of the marriage market.

#### 3.3.1 The Divorce Rule

We assume that the divorce rule  $\varepsilon^*(k, k_m, q)$  maximizes the present discounted value of the spouses:

$$\begin{aligned} & Y^E(k, k_m, q_{-1}) \\ = & \max_{\varepsilon^*} \left\{ \int_q \left[ \int_{-\infty}^{\varepsilon^*(k, k_m, q)} [V_{SM}^R + V_{SF}^R(k)] d\Phi(\varepsilon) \right] df(q|q_{-1}) \right. \\ & \left. + \int_q \left[ \int_{\varepsilon^*(k, k_m, q)}^{\infty} Y^R(k, k_m|q, \varepsilon) d\Phi(\varepsilon) \right] df(q|q_{-1}) \right\} \end{aligned} \quad (2)$$

, where the equal weighting of the spouses follows from the transferable utility assumption. As a convenience, we can write the divorce probability arising from the optimal divorce decision rule as:

$$\pi_{k,k_m}^D(q_{-1}) = \int_q F(\varepsilon^*(k, k_M, q)) d\Phi(\varepsilon) df(q|q_{-1}) \quad (3)$$

### 3.3.2 New Marriages

We can now define the surplus from a new marriage where the bride has  $k$  children as:

$$S(k, 0) \equiv Y^E(k, 0|q_{-1}) - V_{SF}^R(k) - V_{SM}^R$$

. Given that a man has probability  $\omega_0(\phi_k)$  of getting the marital surplus, the *ex ante* net value of a man's prospects in marriage market  $k$  is given by

$$V_{SM}^E(k) = V_{SM}^R + \omega_0(\phi_k) S(k, 0) \quad (4)$$

### 3.3.3 Singles

Recalling the definition of the value functions, we can write the *ex post* continuation value for single men as:

$$V_{SM}^R = \max_k \{u_{SM} + \beta V_{SM}^E(k)\} \quad (5)$$

. Similarly for single women with  $k$  children, the *ex ante* net value of entering the marriage market is:

$$V_{SF}^E(k) = V_{SF}^R(k, \phi) + [1 - \omega_0(\phi_k) - \omega_1(\phi_k)] S(k, 0) \quad (6)$$

. Letting the function  $EU^{SF}(k) = (1 - \pi_k^{SF}) u_{SF}(k) + \pi_k^{SF} u_{SF}(k+1)$ , then If  $\pi_k^{SF}$  is the optimal fertility probability, the *ex post* continuation values for single women are:

$$V_{SF}^R(k) = EU^{SF}(k) - \Theta(\pi_k^{SF} | \widehat{\pi}_k^{SF}) \quad (7)$$

$$+ \beta [(1 - \pi_k^{SF}) V_{SF}^E(k) + \pi_k^{SF} V_{SF}^E(k+1)] \quad (8)$$

## 3.4 Fertility Decisions

Having a child changes the state of the household. The net benefit of having a child therefore depends on the forecast of the probability of marital transitions, which for married couples depends on the current value of  $q$ . For any function  $g(k)$ , let  $\Delta g(k) = g(k+1) - g(k)$  represent the effect of having

one more child. Let the optimal fertility choices be denoted by  $\pi_k^{SF}$  for single women, and by  $\pi_{k,k_m}^{MF}$  for married couples. At interior solutions, the first-order conditions for fertility are then

$$\Theta'(\pi_k^{SF}) = \Delta u_{SF}(k) + \beta(1 - \delta) \Delta V_{SF}^E(k, \phi) \quad (9)$$

for single women with fewer than  $K$  kids, and

$$\Theta'(\pi_{k,k_m}^{MF}) = \Delta u_M(k, k_m) + \beta [\Delta Y^E(k, k_m | \phi, q)] \quad (10)$$

for married women. Corner solutions (maximum fertility) are possible, as we will assume that the technology can only be used to reduce fertility.

### 3.5 Market-Clearing: Determination of $\phi_k$

As in the example, the current model has two types of equilibria; one where the free-entry condition binds and single men are in excess supply, and one where the resource constraint binds, and the free-entry condition does not.

Let  $\mathcal{M}$  be the set of active marriage markets of type  $k$ . We define the men's participation constraint as the requirement that in any active sub-market of type  $k$ , men receive the same expected payoff:

$$V_{SM}^E(k, \phi) = V_{SM}^R(\phi) + \gamma \quad \forall k \in \mathcal{M} \quad (11)$$

If men were to receive less than this in sub-market  $k$  then entry would be suboptimal, so the sub-market would be inactive. If men were to receive more, then more men would enter sub-market  $k$ , implying that  $\phi$  was not the equilibrium vector.

The free-entry condition is that the value of being a single man in the equilibrium with marriage markets is at least as great as the value of autarky:

$$V_{SM}^R(\phi) \geq V_{SM}^A \quad (12)$$

If in equilibrium free-entry condition binds, then the continuation value  $V_{SM}^R$  equals the autarky value  $V_{SM}^A = \frac{u_{SM}}{\beta(1-\delta)}$ . Now suppose that single men strictly prefer entry into active marriage markets. Another way to think of this is that there is excess demand for husbands; the supply constraint binds. This constraint is

$$\sum_k N_M(k) \leq N_M \quad (13)$$

Given our assumption that the match surplus is declining in  $k$ , there is some  $k^* \in \{0, 1, \dots, K\}$  such that in equilibrium  $\phi_k > 0$  if  $k \leq k^*$  and  $\phi_k = 0$  otherwise. Using the definition of queue length, we can write the supply constraint as:

$$\sum_{k \leq k^*} \phi_k N_F(k) = \sum_{k \leq k^*} N_M(k) = N_M$$

. Since the market-clearing implies that  $\phi_k$  is decreasing in  $V_{SM}^R$ , then it is easy to solve for equilibrium by increasing  $V_{SM}^R$  from the autarky level until this constraint holds with equality. In practice the fact that annual marriage rates are low and the supply of men and women roughly equal overall, implies that some men must be sitting out of the market, so the first sort of equilibrium is the more relevant.

### 3.6 Equilibrium

We summarize the model with a formal definition of the stationary equilibrium.

**Definition 1** *A stationary equilibrium of the directed-search marriage market with a maximum kids  $K$  consists of the following objects: a list of decision rules for fertility  $\pi_k^{SF}, \pi_{k,k_m}^{MF}$ , and divorce rules  $\varepsilon^*(k, k_m, q)$  a list of ex-post value functions  $V_{SM}^R$  and  $V_{SF}^R(k)$  for singles and  $Y^R(k, k_m, q)$  for married, a list of distributions  $N_F(k), M(k, k_m, q)$ , a list of queue-lengths  $\{\phi_k\}_{k=0}^K$  for and a law of motion  $T(k, k_m, Q)$  for the distributions. This list must satisfy the following conditions:*

1. *Optimality:*

- (a) *The fertility decision rules are solutions to the individual optimization problems (9) and (10),*
- (b) *the divorce thresholds  $\varepsilon^*(k, k_m, q)$  set the marriage surplus to zero.*
- (c) *For each  $k$ , the value functions solve the the system of equations (1),(5),*

2. *Market-clearing: the market tightness vector  $\{\phi_k\}_{k=0}^K$  satisfies these conditions:*

- (a) *Feasibility: the supply constraint (13) is satisfied.*
- (b) *Free entry: the free entry condition (12) is satisfied for all markets where  $\phi_k > 0$*

3. *Aggregation: The laws of motion of the distributions satisfy:*

- (a) *Consistency: The laws of motion are generated by the individual decisions.*
- (b) *Stationarity: The distributions are the fixed points of their laws of motion.*

## 4 Solving the Model

The decision rules in market  $k$  depend on the other markets  $k' < k$  only through the values of  $\phi_k$  and  $V_{SM}^R$ , while those with  $k' > k$  also determine the value of transiting to a new state. Therefore we can solve the asset equations for each level of  $k$  separately, conditional on conjectured values of  $\{\phi_k, V_{SM}^R\}$ , by backwards induction from  $k = K$ . Given the complete system of decision rules, we then solve for steady-state distributions, starting from  $k = 0$ , since the distribution at level  $k$  is not affected directly by the system at  $k' > k$ . This yields new values of  $\{\phi^0 \dots \phi^K, V_{SM}^R\}$  implied by the market-clearing conditions. We then repeat the procedure using the new values until they converge.

This sequential procedure works because the only transition we allow in  $k$  is to increase by one. To ensure that this procedure converges quickly, we hold fixed the markets that are active. Letting  $k^* \leq K$  indicate the highest market that is open, we start from  $k^* = 0$  and apply the solution procedure for successively higher values of  $k^*$  until we get either  $k^* = K$  or  $\phi_{k^*+1} = 0$ .

### 4.1 Asset Equations

To solve the asset equations for a given level of  $k$ , we solve for the policy rules  $\{\pi^D(k, k_m, q), \pi^F(k, k_m, q)\}_{q=1}^{n_q}$ , and the surplus vector  $\{S(k, 0, q)\}_{q=1}^{n_q}$ . Suppose that marriage market  $k$  is active. Let's assume that we know the value functions for  $k + 1$ , the fertility and divorce probabilities for  $\{(k, q_i)\}_{i=1}^{n_q}$  and the *ex post* value  $V_{SM}^R$  of being a single male.<sup>8</sup> These assumptions allow us to write the asset equations relevant to the marriage market for women with  $k$  children as the following linear system:

$$\begin{bmatrix} V_{SF}^E(k) \\ Y^E(k, 0, q_1) \\ \dots \\ Y^E(k, 0, q_{n_q}) \end{bmatrix} = A_{1k} \begin{bmatrix} V_{SF}^E(k) \\ Y^E(k, 0, q_1) \\ \dots \\ Y^E(k, 0, q_{n_q}) \end{bmatrix} + A_{0k} \quad (14)$$

. The elements of  $A_{1k}$  and  $A_{0k}$  are derived in the appendix. Note that this system is independent of the value of being a family with  $k$  children and  $k_m > 0$ , because those outcomes have zero probability for these women. However, the value of a  $(k, k_m > 0)$  family depends on the value of being single with  $k$  children, so with the solution to (14) in hand we then solve a second, smaller linear system for the values of these families:

$$\begin{bmatrix} Y^E(k, k_m, q_1) \\ \dots \\ Y^E(k, k_m, q_{n_q}) \end{bmatrix} = B_{1k} \begin{bmatrix} Y^E(k, k_m, q_1) \\ \dots \\ Y^E(k, k_m, q_{n_q}) \end{bmatrix} + B_{0k} \quad (15)$$

The elements of  $B_{1k}$  and  $B_{0k}$  are derived in the appendix.

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<sup>8</sup>Note that the transitory shock  $\varepsilon$  is not part of the state vector.



## 4.2 Distributions

Using the marriage and fertility decision rules derived above, we can easily infer the laws of motion and then compute the steady-state distributions of the household types. Within each level of  $k$  we solve separately for the households with  $k_m = 0$  and those with  $k_m > 0$ .

Let the next-period mass of the singles and married at each state be given by  $N'_F(k)$  and  $M'(k, k_m, q)$ , respectively. We show in the appendix that we can write the law of motion  $T(k, 0, Q)$  of the distribution of singles and marriages with  $k_m = 0$  as the linear system:

$$\begin{bmatrix} N'_F(k) \\ M'(k, 0, q_1) \\ \dots \\ M'(k, 0, q_{n_q}) \end{bmatrix} = C_{1k} \begin{bmatrix} N_F(k) \\ M(k, 0, q_1) \\ \dots \\ M(k, 0, q_{n_q}) \end{bmatrix} + C_{0k} \quad (16)$$

, where the elements of  $B$  and  $b_1^k$  are derived in the appendix. This linear system is easily solved for the stationary values.

For any  $k > 0$ , we first have to solve for the stationary distributions of married couples with  $k_m > 0$ . This is the fixed point of the linear system defined by  $T(k, k_m, Q)$ :

$$M'(k, k_m, Q) = D_{1k, k_m} M(k, k_m, Q) + D_{0k, k_m} \quad (17)$$

Because any increase in  $k_m$  entails an increase in  $k$ , the law of motion for each different value of  $k_m$  forms a separate linear system of equations that depends on behavior at  $k - 1$ ; unless  $k_m > 0$ , the behavior of women with  $k$  children is not required to solve these equations.

## 4.3 Market-Clearing

The equilibrium solution can be reduced to the appropriate choice of the ex post value for men,  $V_{SM}^R$ .

Using equations (4) and (11), it is clear that

$$V_{SM}^R(\phi) - V_{SM}^A = \omega_0(\phi_k) S(k, 0) - \gamma$$

Taking our guess on  $V_{SM}^R$  as given, we can easily solve this for the queue length as a function of the surplus:

$$\phi_k = -\log(\omega_0) = \log(V_{SM}^R - V_{SM}^A + \gamma / (S(k, 0)))$$

There is no guarantee that the free-entry condition is solved by a positive  $\phi_k$  in every market. In those markets where the solution would require  $\phi_k < 0$ , the non-negativity constraint binds; these markets do not operate, as men prefer to enter another market or to remain single. In any case, this equation

implies the queue-length vector, so we can compute the distributions and hence the supply and demand for single men. We start by setting  $V_{SM}^R = V_{SM}^A$ , solving the model, and checking for excess supply of single men. If this is positive we are done; otherwise we increase  $V_{SM}^R$  until excess supply equals zero.

## 5 Calibration

We now put the model to work to measure the importance of marriage-market prospects for unmarried fertility and to assess the effects of different types of shocks to marital behavior. We calibrate the model to the 1970s because, as we saw in the empirical analysis (Table 3), single women with children were much less likely to marry, an effect that has faded away in the 1990s data. The procedure consists of choosing a set of statistical targets, choosing functional forms, setting values for the parameters of the model, solving the model for the given values and functional forms, simulating a cohort of women in the model, and comparing model statistics from the simulation to the statistical targets. The parameters can be divided into three sets; “fixed” parameters, whose values can be pinned down directly from empirical observations or convention, “normalized” parameters, those that will be held fixed at arbitrary values, and “free” parameters, whose values will be set so as to minimize the distance between the targets and the relevant model statistics.

The statistical targets are the probit predictions, from Table A1, of the birth and marriage probabilities for unmarried women at age 25, and the birth and divorce probabilities for married women at the same age. These targets are listed in Table 5(a). Because of the small sample size (2379 unmarried women in the 1973 NSFG) we must be careful to avoid basing targets on very small cells of respondents, such as divorced women with 2 children, or worse, divorce rates of families with step children. We therefore use the aggregate divorce rate and limit the targets for marriage and divorce to women with at most two children.

### 5.1 Functional forms

The stochastic process governing marital quality consists of two components: a persistent component  $q$  that follows a Markov chain with parameters  $\rho_q, \sigma_q$ , and a transitory component that will be log normal with mean zero and variance parameter  $\sigma_\epsilon$ .

For non-sterile women, the probability that a child will arrive next period is assumed to be a declining function of contraceptive effort, which is modeled as a utility cost  $\Theta(\pi_i^F)$  to the household. Therefore those who prefer to have a child will exert zero effort. Let the fertility probability at zero effort, for a woman of marital status  $i$  be  $\hat{\pi}_i$ . For fertility-cost parameters  $(\eta_1, \eta_2)$ , the effort-probability

frontier is given by:

$$\Theta(\pi_i^F) = \eta_1 \left( \frac{1}{\max\left(0, \frac{\pi_i^F}{\hat{\pi}}\right)^{\eta_2}} - 1 \right) \quad (18)$$

. The main implications of this functional form are: 1) effort smoothly approaches zero as fertility approaches the maximum, and 2) effort tends to infinity as the fertility probability tends to zero, and 3) the elasticity of the fertility probability is decreasing in  $\eta_2$ .

The utility flows generated by different household types are parameterized as linear functions of the number of children. Thus married households without children receive utility flow  $\alpha_M$  and single households without children receive utility flow  $\alpha_S$ . The first child increases utility by  $\alpha_W^0$ , for single women, and by  $\alpha_W^0 + \alpha_H^0$  for married couples. The arrival of additional children increases utility of single households by  $\alpha_W^1$  and that of married, if the child arrives inside the marriage, by  $\alpha_W^1 + \alpha_M^1$ . The marginal effect of an additional child born previous to the marriage is  $\alpha_W^1 + \alpha_M^2$ ; this represents the utility penalty associated with step children. The interpretation of the differentials  $\alpha_M^j$  between married and single life include the impact of children on the husband, as discussed in the introduction, but also allow for women's preferences for raising children in a married household, perhaps for other reasons than the husband's preferences, such as increased resources or some less tangible benefit of having the father present.

## 5.2 Parameters

### 5.2.1 Fixed Parameters

As in Regalia and Ríos-Rull (1999), the probability  $\delta$  of exiting the active state is set so as to replicate the average number of years a woman is fecund. Sommer (2008) reports the fraction of women who are fecund at each age between 16 and 44, using an interpolation of estimates by Trussell and Wilson (1985). This results in a total of 20.45 fecund years per woman, so we set  $\delta = 0.0489$ .

These fixed-parameter values are shown in Table 4(b). We set  $\beta = 0.96$ , the standard value for annual frequencies in the macroeconomics literature; in models with savings, this value ensures that the risk-free interest matches the US long-run average of 0.04.

The curvature  $\eta_2$  in the effort-fertility frontier is set to 0.1. See the discussion section below regarding the difficulty of identifying an empirical analog for this parameter. At an interior solution, letting  $\Delta V$  be the (negative) gain from having a child, the optimal fertility is given by

$$\pi^* = \hat{\pi}^{\eta_2} \left[ \frac{-\eta_1 \eta_2}{\Delta V} \right]^{\frac{1}{\eta_2 + 1}} \quad (19)$$

. This means the elasticity of fertility with respect to  $\Delta V$  declines to zero as  $\eta_2$  increases.

### 5.2.2 Normalized parameters

The values of some parameters are fixed arbitrarily. Thus the divorce cost is set to 2, and the variance parameters  $(\sigma_q, \sigma_\epsilon)$  of both the persistent and the transitory quality shocks are set to 1. This implies the support of the persistent quality is  $[-1.62, 0, 1.62]$ . The utility flow of married couples with no kids is set to 2, and the entry cost  $\gamma$  to 1. These are classed as normalizations, because the values can be changed without affecting the overall results; the calibration procedure will reset the values of the free parameters to restore the behavior of the model. The probability of marriage, conditional on matching, also appears in the table. This was implicitly set to  $p_z = 1$  in the theory discussion, and remains so in the calibration. Lower values will be used in the comparative statics section below to identify the impact of frictions in the fertility decisions. The values of these parameters are listed in Table 4(a).

### 5.2.3 Free Parameters

The eight remaining parameters, also listed in Table 4(a), are set in the standard way: we identify a list of target statistics from the 1973 NSFG, and the parameter values are set so as to minimize the distance between these targets and the corresponding model statistics. The targets consists of marriage and birth rates by marital status and number of children, as well as the average divorce rate, for the 1969-72 period in the survey. We use the predicted values for 25-year old women who have graduated high school but not attended college, based on the Probit regressions reported in the empirical section of the paper (Tables 3 and A1 for unmarried women, and Tables A2 and A3 for married). These are compared to the simulated statistics for women in their seventh year, which, since we label the initial year age 18, corresponds to age 25.

## 5.3 Simulation

Taking the parameters as given, we solve for the model's stationary steady state using the recursive strategy outlined in the previous section. For computing convenience, we keep the state space of the model relatively small: we set  $K = 3$  and  $N_q = 3$ . The computation starts from an initial guess of the market-clearing vector  $\phi$  and the value  $V_{SM}^R$  of being a single man, then solves for the optimal behavior conditional on these guesses. The optimal behavior generates, via the implied steady-state distributions, a new market-clearing vector  $\phi$ , which allows us to update the value  $V_{SM}^R$ . We repeat the procedure until convergence.

The model statistics are generated from the simulation of a cohort of women, starting at age 18. The model statistics therefore reflect the model's stationary steady state at a given set of parameter values. The simulation is in turn based on the decision rules generated from the model's equilibrium

decision rules, evaluated at the current parameter-value guesses. Although the model solution generates steady-state distributions of women, these are for active women only; inactive women are not tracked by the model, hence the recourse to simulation.

In the simulation, women start out active at age 18 and eventually become inactive (sterile), according to the constant-hazard process described in the model section. In this way the simulation translates the model solution into a distribution by age, even though age is not a state variable in the model. The cohort size is set to  $N = 10,000$  women, and the simulation follows each woman for 27 years, to correspond to the 18-44 age group. The initial conditions at entry are marital status and number of children. These are set to match the average for 18 year olds in 1973; according to the March CPS, 14% of women aged 18 were already married. For each woman, the realizations of the stochastic processes governing marriage, fertility, divorce are given by 27 iid draws of a uniform random variable of dimension five<sup>9</sup>. The aggregate statistics are then computed by pooling the observations over the entire population, over all ages, while the targets are based only on the 25 year-olds.

## 6 Results

The benchmark values for the eight free parameters are shown in Table 4(a). As discussed in the functional forms section above, the marginal utility of children is negative, in order to generate effort in reducing fertility. The “Men’s utility” less parameter is positive; this simply means that married couples are less averse to fertility than are single women. The negative effect of step children of course is required to match the lower marriage rates of single mothers; otherwise the fact that they are worse off as singles than non-mothers would generate a higher marriage surplus and hence higher marriage rates. The persistence of marriage quality is set to 0.524; this was required in order to match the divorce rate, given that the divorce cost was fixed. The utility bonus for being single was -0.511; this is the main systematic utility gain from marriage, which is otherwise driven by the match-quality process, which delivers a utility bonus of 1.62 to good marriages.

The first column of Table 4(b) shows the nine targets, the second the model statistics (the third and fourth columns, labelled ‘1995’, will be discussed in the next section). The average deviation between model and target statistics is 8%, the largest being the birth rate for single mothers, which is 5.6% annually in the NSFG and 6.5% in the model. The birth rate for unmarried women without children is 1.9% in the data and 1.7% in the model, a 10% deviation. There is no reason to believe that a closer match is not possible; but previous calibrations (not reported) with reduced precision delivered similar

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<sup>9</sup>This corresponds to one realization for each of the following events: getting married, marriage quality, divorce, fertility, and exit from active status.

results, so it seems reasonable to proceed with these values.

The aggregate statistics for the full 18-44 group are shown in Table 5. There are four reasons one that these can differ between model and data, even with a perfect calibration: 1) the model calibration does not target women with more than one child, 2) the model does not allow for fertility of women with 3 or more children, 3) the calibration ignores women who are not aged 25, and 4) the simulation assumes a stationary distribution, while unmarried behavior in the early 1970s, as shown by Ventura (2009), was already changing relative to 1960 and earlier. Nevertheless, the first two columns show that the model delivers reasonably accurate values for the fraction of women married (74%) and the average birth rate (15%). It does sharply underestimate the share of fertility due to unmarried women (6% instead of 10%), the fraction of families with kids from two or more fathers (2% instead of 6%) and the fraction of children in single-mother households (6% instead of 13%); overall the model delivers a picture of an economy dominated slightly more than reality by nuclear families.

Table 6 shows the decision rules for fertility and the marriage probabilities, conditional on being active. The effects of marriage quality on fertility are seen to be small. Married couples with no children always want children, so they have the maximum fertility rate, 0.3. The shaded squares in panel 6(a) show families with no step children; these always have lower fertility than same-size families with step children; for instance active families with two children have a birth rate of 0.19 if none of the children are the husband's, but this declines to 0.12 as the number of the husband's children increases. In table 6(b) we see a marriage rate of 0.41 for active women without children; given that they have no children, women are much less likely to be active in the simulation, so high marriage rates are required. Women with one child marry at rate 0.18 and with two children at rate 0.096. The table also shows the impact of fertility on marital prospects; the surplus is 1.79 for women without kids, declining to 1.21 for women with two kids; furthermore the (unconditional) probability that the woman gets the surplus is 10% for women without children and declines to 0.04% for women with two kids. The marriage rates of women with more than two children are zero, an artifact of setting  $K=3$ , which reduces the gains from marriage for these women. Since unmarried women with more than two children are such a small fraction of the population in the 1970s, this simplification seems justified.

## 6.1 Comparative Statics

To clarify the behavior of the model, we now explore the impact of varying some of the free parameters. The premise of the model is that marriage and fertility decisions are connected through marital prospects, so it is useful to see how fertility responds to parameters that affect marriage directly, and, conversely, how marriage rates respond to parameters that affect fertility decisions.

In Figure 5, we increase the slope  $\eta_2$  of the contraception-effort frontier from 0.1 to 1.2 (the benchmark value being 0.39). One reason that this is interesting is that contraception effectiveness did change over time with the advent of the contraceptive pill in 1960 and liberalization of abortion in the 1970s. Panel (a) shows that the effect of reducing the marginal cost of contraception is, not surprisingly, to sharply reduce the unmarried fertility rate. In the context of the model, this would not be a good explanation of rising unmarried fertility. Panel (b) shows the aggregate birth rate declining, again not surprising. Increasing contraceptive effectiveness does however generate marital decline. In Panel (c) we see significant effects on the marriage hazard. Finally panel (d) shows that the unmarried share of birth rates fall as  $\eta_2$  falls, as does the fraction of mothers who are unmarried, but the absolute magnitude of the response is very small, ranging from roughly 1% to 5%.

In Figure 6 we increase the utility bonus  $\alpha_S$  associated with single life from -0.58 to -0.46; the benchmark value is 0.51. Obviously this will generate a decline in marriage rates, and this is shown in Panel (c), where marriage rates decline from above 0.3 to zero. The critical observation is that the unmarried share of fertility increases, as shown in panel (d) from 0.025 to about 0.94. This is partly due to a decline in the share of women who are married, but, as in the data, the birth rate to single women rises significantly too. Panel(a) shows the birth rate to unmarried women with no children rises from zero to 0.08, and that for mothers from 0.03 to 0.08. The annual birth rate nevertheless declines, as unmarried fertility remains below that of married women.

Figure 7 shows the effect of varying the divorce cost  $d_C$ . In the benchmark model this is set to 2.0. It is generally accepted that divorce costs fell from the 1960s to the 1970s, but it is not obvious how the 1990s compare to the 1970s. We see in panel (a) that reducing divorce costs causes unmarried birth rates to rise slightly for unmarried mothers, from 0.018 to 0.22, but this is partially offset by a stronger decline in the birthrates of unmarried mothers, from 0.06 to 0.04. Overall, annual birthrates decline quite sharply, as shown in panel(b). This is driven by the decline in marriage rates of non-mothers, as shown in panel (c), from 0.25 to 0.22. The share of unmarried fertility in birthrates therefore increases overall, as shown in panel (d), as does the unmarried fraction of single mothers.

Finally in Figure 8 we see the effect of reducing  $p_Z$ , the probability that marriages are allowed at a given moment, which we call the marriage completion rate. As  $p_Z$  falls below 1, the matching frictions increase relative to the benchmark model. This is a different way to index the value of marital prospects. In panel(a) as the friction increases, unmarried birth rates increase; the effect is strongest for non-mothers of course, as they are the ones for whom these prospects are most important, so their birthrates converge to those of the mothers. Nevertheless, the aggregate birth rate falls, as the fraction of women in marriage declines to zero by  $p_Z = 0.2$  as it is no longer worthwhile for men to pay the entry cost. The birth effects level off as soon as this point is reached. It is clear from this experiment

that marital prospects are the main mechanism in the model for explaining the fact that unmarried mothers have much higher birth rates than non-mothers.

## 6.2 Age Profiles

While age is not a state variable in the model, it is possible of course to track age in the simulation, and so the lifecycle profiles over this age interval can be traced and compared to the analogous profiles in the data. Figure 9 shows the marriage and birth age profiles for unmarried women based on the 1973 NSFG, which were first shown in Figure 2. It also shows the profiles from the Benchmark simulation. The model profiles for the marriage rate of single mothers and for the birth rates of unmarried non-mothers are remarkably close to the NSFG profiles, but panel (a) shows that the model series for marriage rates of unmarried non-mothers starts too high and declines too quickly. This suggests that abstraction from aging is not the problem in itself, but rather it is abstraction from an activity that competes with matching at early ages, such as human-capital accumulation.

The birthrate profiles are relatively close to the data. In panel (b) we see that birthrates to unmarried mothers fail to fall quickly enough but the model series for non-mothers actually lies on the NSFG series for about half of the profile. Panel (c) shows that for married couples the series for both mothers and non-mothers are relatively close.

Overall, the model does a fine job of approximating the age profiles for birth rates as well marriage rates for unmarried mothers. Extending the model to include education would certainly help depress the marriage rates of the younger women, but may make it harder for the model to match the relatively high unmarried birth rate for these women. On the other hand, extending the model to allow for pre-marital sex could delay marriage while maintaining the high rate of unmarried births.

## 6.3 Experiment: Marriage Prospects

An important aspect of the model design is that it is possible to vary taste parameters that affect marriage and divorce rates without directly affecting the utility of having children. In this section we exploit this feature by resetting three parameters to match the 1990s values of three targets: the marriage rates of women without children, the marriage rate of women with one child, and the average divorce rate. As with the benchmark calibration for 1973, the targets consist of the predicted values for age-25 women without college education, derived from the probit estimates in Table 3. We then ask to what extent the change in marital prospects associated with declining marriage rates can account for the observed increase in the birth rates to unmarried women.

The three parameters are: the utility of single life, the effect of step children on the married utility,



and the cost of divorce. The new values are shown in column 2 of Table 4(a). and the target and model statistics are shown in columns 3 and 4 of Table 4(b). The shaded cells in the table correspond to statistics that were not targets. To hit the 1995 targets required the negative effect of step kids on married utility to shrink from  $-.05$  to  $-.008$ , the contribution of single life to utility to grow from  $-0.511$  to  $-0.487$ , and the cost of divorce to increase 11% from 2 to  $2.22^{10}$ .

The effect on fertility of unmarried women is striking: birth rates for non-mothers increase from 0.017 annually to 0.045, mirroring the increase observed in US data.

Some of the other changes in non-targeted statistics also appear remarkably similar to their counterparts in the data: marriage rates of women with two children, for example, rise from 8% annually to 12% annually in both model and data. The birth rate for married women with 1 child rises from 19% to 21% in both model and data. Other non-targets, such as the birth rates to married women with no children or two children, remain constant in both model and data. Where the model might have done better is in accounting for the rise from 5.6% to 9% in unmarried birth rates for women with one child; in the model this declines slightly, from 6.5% to 5.3%.

Table 5 shows that the changes in the aggregate statistics for the model also mirror the changes observed in the US. The fraction of women married for instance fell from 74% to 48% in the model, and to 53% in the data. The fraction of children in single-mother households rose from 13% to 22% in the data, and from 6% to 28% in the model. The share of fertility accounted for by unmarried women rose from 10 to 30% in the data, and from 6% to 26% in the model. The fraction of families with both step-children and father’s children rose from 6% to 10% in the data and from 2% to 8% in the model. Thus the model results suggest changing marital patterns could potentially explain all of the rise in unmarried birth rates.

## 7 Discussion

One sense in which the design of the experiment above may have biased the result towards finding a large impact of marital prospects is that we have treated all women as identical *ex ante*. It may be that unmarried women who have children were less likely than others to marry anyway. Does omitting this heterogeneity exaggerate the impact of marriage prospects? Note that even in this argument the fertility arises from weaker marital prospects. Therefore it is not clear whether our model is exaggerating or

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<sup>10</sup>The reason divorce costs increase is that when marriage rates decline, divorce rates, if marriage quality is iid, have to rise. The calibrated persistence (0.55) appears insufficient to overturn this effect through higher quality of marriages. It may be that increasing the cardinality of the support of the quality shock (currently set at 3) may help to increase marriage quality, but we have not explored this.

merely simplifying the channels through which the effects occur. The estimates of Rosenzweig (1999) suggest that the direct effects of marital prospects on fertility are large, but his model is quite different, and so it is not possible to compare his values with the results here. We eagerly await the results of future research to shed light on this issue.

The critical parameter for generating birth-rate responses to shifting marital prospects is the curvature parameter  $\eta_2$  in the effort-fertility frontier. It should be stressed that in reality this frontier comprises a number of margins that are not modeled explicitly here; most importantly perhaps the margins for sexual activity, for contraceptive-method choice, and for abortion. Therefore “effort” corresponds to, *inter alia*, foregone utility from sexual abstention, the cost and disutility of contraception, and the risks and utility costs associated with abortion.

This paper found that marital prospects could explain why fertility of unmarried women was so low in the 1970s, provided that  $\eta_2$  was sufficiently close to zero. Whether this is a plausible representation of the actual effort-fertility frontier would seem to require more explicit modelling of the fertility decision, a task that is outside the scope of this paper. However the current model could serve as a basis for such modeling. In Kennes and Knowles (2011) for instance, the sex margin for unmarried people is added to the model, so that unmarried women choose whether to be sexually active; those who opt for sex then choose how much contraceptive effort to exert. That effort-fertility frontier is therefore easier to measure than the one in this paper, because it does not include the sexual activity margin. Building on the theory presented the current paper therefore seems to be a logical route to answering the questions raised by the transformation of the behavior patterns of unmarried women.

That an important component of this effort is associated with the actual use of contraception is suggested by a host of results from the health literature on birth control. It is well known that the theoretical effectiveness of birth control methods is much higher than their measured effectiveness in use. Trussell, Hatcher, Jr., Stewart, and Kost (1990) for instance estimate the failure rate of the pill under “consistent and correct use” to be one tenth of one per cent in the first year of use, while that of the condom would be 2% and that of the diaphragm 6%. Reviewing recent studies, however, they find typical condom failure rates of 12%, of the pill of 3% and diaphragms 18%. It turns out that users make mistakes, to which the public-health response has been intensive campaigns to ensure women are well-informed regarding usage. In a recent editorial column in the journal *Contraception*, Glasier and Shields (2006) argue from the results of a host of empirical studies that information has no impact on effectiveness. User “mistakes” are therefore much more better explained by the theory that “consistent and correct use” requires user effort. If avoiding mistakes requires effort, the rate of effectiveness will depend, *inter alia*, on the incentives to avoid pregnancy.

It is now nearly twenty years since the NSFG wave of 1995 studied in the paper. Since that time,

cohabitation has become increasingly important as an alternative or a route to marriage. This suggests another route for extending the analysis presented here, by adding a margin for co-habitation, perhaps even a separate set of matching markets. The timing of events suggest that far from being a cause of the decline of marriage, cohabitation is rather a symptom of that decline which could be analyzed by putting more structure on the options for unmarried women in the model.

## 8 Conclusions

The results demonstrate that the greater importance of marital prospects in the 1970s than in the 1990s may be sufficient to account for the lower birth rates of unmarried women at the time. We exploited the fact that unmarried mothers marry at a lower rate than non-mothers to calibrate the effect of marital prospects in the matching model. Although the exercise sheds little light on *why* marital patterns changed, the comparative statics exercises suggest an important role for shocks that increase the utility of single life. A possible candidate explanation for this change, the arrival of highly-effective contraception for single women, is explored in Kennes and Knowles (2011), which extends the current theory to the analysis of sexual activity of unmarried people.

Our quantitative results are not meant to be definitive but rather should be taken as illustrations of the usefulness of our approach; the presence of children has been ignored in most analysis of the marriage market, but our results confirm that the interaction between marriage and fertility is significant. The contribution of our model is to allow repeated opportunities to remarry and to have children; to get there we abstracted from important features explored in related papers, such as aging, human-capital investment in children or the impact of means-tested government transfers. There are also important features of marriage, such as the margin between cohabitation and marriage, that are ignored by both the current paper and the bulk of the related literature. However it is clear that the approach used here can be extended to deal with these and other features of marriage and fertility.

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# APPENDIX

## A Solving the Asset Equations

In this section we derive the coefficients of the linear asset-equation systems that define the value functions. Recall that  $q \in Q = \{q_1, \dots, q_{n_q}\}$  and that the probability of the first shock in a marriage being  $q_i$  is  $f(q_i; \hat{q})$ , where  $\hat{q} \in Q$  is the same for all new marriages.

We write the divorce probability, before the current realizations  $(q', \varepsilon')$  of marriage quality are known, as:

$$\pi_{k,k_m}^D(q) = \sum_{q' \in Q} f(q'; q) \Phi(\varepsilon^*(k, k_m, q'))$$

, where  $\varepsilon^*(k, k_m, q')$  refers to the optimal divorce rule defined in the model section of the paper.

The probability that a single woman with  $k$  children marries is

$$\mu_k \equiv p_z [1 - \omega_0(\phi_k)] [1 - \pi_{k,0}^D(\hat{q})]$$

, where  $p_z \in [0, 1]$ , a parameter denoting the probability that matched couples are allowed to marry, generalizes the model slightly to allow for market-frictions. When we solve the level- $k$  system for  $k < K$ , we assume that we already know the solution for the  $k + 1$  system of asset equations. The system at  $k = K$  is relatively easy to solve because with fertility assumed to be zero, there are no transitions to higher  $k$ .

### A.1 Single Female

Let the probability that a single female obtains the marriage surplus be

$$p_S(k) \equiv p_z [1 - \omega_0(k) - \omega_1(k)] [1 - \pi_{k,0}^D(\hat{q})]$$

The *ex ante* value of being a single female with  $k$  kids is:

$$V_{SF}^E(k) = V_{SF}^R(k) + p_S(0) S(k, 0)$$

, where  $S(k, 0) = Y^E(k, k_m, \hat{q}) - V_{SF}^R(k) - V_{SM}^R$  denotes the surplus.

By the definition of  $V_{SF}^R(k)$ , we can write :

$$V_{SF}^R(k) = \tilde{d}_1 + \beta(1 - \delta)(1 - \pi_k^{SF})V_{SF}^E(k) \quad (20)$$

, where the terms in  $\tilde{d}_1$  are known<sup>11</sup>:

$$\begin{aligned} \tilde{d}_1 &= (1 - \pi_k^{SF})u_{SF}(k) + \pi_k^{SF}u_{SF}(k+1) \\ &\quad - \Theta(\pi_k^{SF}) + \beta(1 - \delta)\pi_k^{SF}V_{SF}^E(k+1) \end{aligned}$$

. Plugging this back into the definition of  $V_{SF}^E(k)$  we can write:

$$V_{SF}^E(k) = a_{11}V_{SF}^E(k) + a_{13}Y^E(k, 0, \hat{q}) + d_1$$

where

$$\begin{aligned} a_{11} &= \beta(1 - \delta)(1 - \pi_k^{SF})[1 - p_S(\phi_k)] \\ a_{1j} &= \begin{cases} p_S(\phi_k) & \text{if } q_j = \hat{q} \\ 0 & \text{if } q_j \neq \hat{q} \end{cases} \\ d_1 &= [1 - p_S(\phi_k)]\tilde{d}_1 - p_S(\phi_k)V_{SM}^R \end{aligned}$$

## A.2 Child-Less Marriages

Consider a married couple with no children. We write the divorce probability of a marriage with realization  $q'$  as  $\Phi(\varepsilon^*(k, k_m, q'))$  and the *ex ante* divorce probability, given previous realization  $q$ , as

$$\pi_{k,0}^D(q) = \sum_{q' \in Q} \Phi(\varepsilon^*(k, k_m, q')) f(q'; q)$$

. Recall that the divorce cost is denoted by  $\chi$ . The value of a new marriage where the bride already has  $k$  children equals the sum of two components: the value if the marriage stays intact, and the value if the marriage ends:

$$Y^E(k, k_m, q) = \sum_{q' \in Q} [1 - \Phi(\varepsilon^*(k, k_m, q'))] f(q'; q) Y^R(k, K_{M=m} | \phi, q') f(q'; q) \quad (21)$$

$$+ \pi_{k,k_m}^D(q) [V_{SF}^R(k, \phi) + V_{SM}^R - \chi] \quad (22)$$

---

<sup>11</sup> $\Theta$  and  $u_{SF}$  are parameterized functions,  $\pi_k^{SF}$  is conjectured, and  $V_{SF}^E(k+1)$  was solved for in the previous step.

Assuming the marriage survives the divorce stage, the value of the marriage, before the fertility realization is known, is

$$Y^R(k, 0 | \phi, q') = EU^M(k, q') + \beta(1 - \delta) EY^E \quad (23)$$

, where

$$\begin{aligned} EU^M(k, q) &\equiv q + (1 - \pi_{k,0}^{MF}(q)) u_M(k, 0) \\ &\quad + \pi_{k,0}^{MF}(q) u_M(k + 1, 1) - \Theta^{MF}(\pi_{k,0}^{MF}(q)) \end{aligned}$$

represents the expected flow utility this period, and

$$EY^E = [(1 - \pi_{k,0}^{MF}(q')) Y^E(k, 0, q')] + \beta(1 - \delta) \pi_{k,0}^{MF}(q_i) Y^E(k + 1, 1, q')$$

is the expected continuation value.

Using (20), we can write (21) as the sum of the continuation values without births as married and as single plus a pre-determined component  $d_{j+1}$  that consists of period utility flows and the continuation values with births:

$$Y^E(k, 0, q_i) = a_{i+1,1}^{k,k_m} V_{SF}^E(k) + \sum_{q' \in Q} a_{j+1,i+1}^k Y^E(k, 0, q_j) + d_{j+1}$$

where

$$\begin{aligned} a_{j+1,1}^{k,k_m} &= \pi_{k,0}^D(q_j) \beta(1 - \delta) (1 - \pi_k^{SF}) \\ a_{j+1,i+1}^k &= [1 - \Phi(\varepsilon^*(k, k_m, q_j))] f(q_j; q_i) \beta(1 - \delta) (1 - \pi_{k,0}^{MF}(q_j)) \\ d_{j+1} &= \hat{d} + d_{u,j+1}^k + d_{y,j+1}^k + \pi_{k,0}^D(q_{j+1}) (V_{SM}^R - \chi) \end{aligned}$$

and

$$\begin{aligned} d_{u,j+1}^{k,k_m} &\equiv \sum_{q' \in Q} [1 - \Phi(\varepsilon^*(k, k_m, q'))] f(q'; q) EU^M(k, q') \\ d_{y,j+1}^{k,k_m} &\equiv \sum_{q' \in Q} [1 - \Phi(\varepsilon^*(k, k_m, q'))] f(q'; q) \beta(1 - \delta) \pi_{k,0}^{MF}(q_i) Y^E(k + 1, 1, q') \end{aligned}$$

### A.3 The rest of the value functions

Now that we have computed the value system for single women and newly-weds, it remains to compute the values of single men  $V_{SM}^E(k)$  and the values of marriages with husband's children present, ( $k_m > 0$ ). These are straight-forward. First, for the single men,

$$V_{SM}^E(k) = V_{SM}^R + \omega_0(k) p_z [1 - \pi^D(\hat{q})] S(k, 0) - \gamma$$



. We use this to update  $V_{SM}^R$  when solving the model.

Consider an ongoing marriage where the bride already has  $k$  children of which  $k_m \leq k$  are the husband's.

By assumption, we know the solutions for  $k + 1$ , so the only unknowns are  $\{Y^E(k, k_m, q_i)\}_{q_i \in Q}$ , where  $k_m > 0$ .

The system to solve is (15), with coefficients  $B_{1k} = [b_{ij}]$ , where

$$b_{ij} = f(q_j, q_i) (1 - \pi_{k, k_m}^D(q_j)) \beta (1 - \delta) (1 - \pi_{k, k_m}^{MF}(q_j))$$

and  $B_{0k} = [d_i]$ , where

$$\begin{aligned} d_i &= \pi_{k, k_m}^D(q_i) [V_{SF}^R(k, \phi) + V_{SM}^R - \chi] \\ &\quad + EU^M(k, q_i) + \tilde{\beta} \pi_{k, k_m}^{MF}(q_i) Y^E(k + 1, k_m + 1, q_i) \end{aligned}$$

. Note that the term  $V_{SF}^R(k, \phi)$  is known from the solution to the previous equation (14) with  $k_m = 0$ .

## A.4 Distributions

### A.4.1 case 1: $k = 0$

We now derive the coefficients of equation (16) representing the law of motion for the masses of women at  $k = 0$ .

**Single Women** The law of motion for single women with  $k = 0$  includes the exogenous arrival rate  $\delta$  of new singles as well as the flow of newly divorced child-less women:

$$N'_F(0) = \delta + (1 - \delta) (1 - \pi_0^{SF}) \left[ (1 - \mu_0) S_F(0) + \sum_{q \in Q} \pi_{0,0}^D(q) M(0, 0, q) \right]$$

, where  $S'_F(0)$  is the fraction of the population next period consisting of childless single women. We can therefore write the law of motion of the child-less single mass as

$$N'_F(0) = \delta + a_{11} N_F(0) + \sum_{q \in Q} a_{1, q+1} M(0, 0, q)$$

, where

$$\begin{aligned} a_{11} &= (1 - \pi_0^{SF}) (1 - \delta) (1 - \mu_0) \\ a_{1, q+1} &= (1 - \pi_0^{SF}) (1 - \delta) \pi_{0,0}^D(q) \end{aligned}$$

**Married Couples** The flow out of the married childless state includes both fertility and divorce, while the only flow is from marriage of childless singles:

$$M'(0, 0, q_j) = \kappa_0(q_j) \left[ f(q_j, \hat{q}) p_z (1 - \omega_0) N_F(0) + \sum_{q \in Q} f(q_j; q) M(0, 0, q) \right]$$

where  $M'(0, 0, q')$  is the fraction of women next period who are both childless and married, and

$$\kappa_0(q) \equiv (1 - \delta) [1 - \Phi(\varepsilon^*(0, 0, q))] (1 - \pi_{0,0}^{MF}(q))$$

. We can therefore write the law of motion of the mass of child-less marriages as

$$M'(0, 0, q_j) = a_{j+1,1} N_F(0) + \sum_{i=1}^{n_q} a_{j+1,i+1} M(0, 0, q_j)$$

where

$$\begin{aligned} a_{j+1,1} &= \kappa_0(q_j) p_z (1 - \omega_0^0) f(q_j, \hat{q}) \\ a_{j+1,i+1} &= \kappa_0(q_j) f(q_j, q_i) \end{aligned}$$

. Let  $C_{1k} = [a_{i,j}]$ . The vector of constants (16) is  $C_{0k} = [\delta, 0, \dots, 0]$ .

#### A.4.2 case 2: $k > 0, k_m > 0$

Once the system at  $k - 1$  is known, it is easy to compute the steady-state distribution for  $M(k, k_m, q)$  with  $k_m > 0$  as the fixed point of equation (17). This is particularly easy for  $k_m > 1$  because the only inflow is from  $M(k - 1, k_m - 1, q)$ , whereas for  $k_m = 1$ , we must also allow for inflows from  $N_F(k - 1)$ . To represent this inflow, we define a pre-determined term  $g_{k,k_m}^0(q_j)$ :

$$g_{k,k_m}^0(q_j) = \begin{cases} \kappa_{k,k_m}^0(q_j) f(q_j, \hat{q}) p_z (1 - \omega_0^k) N_F(k) & k_m = 1 \\ 0 & k_m > 1 \end{cases}$$

, where

$$\kappa_{k,k_m}^0(q_j) = (1 - \delta) [1 - \Phi(\varepsilon^*(k - 1, k_m - 1, q_j))] \pi_{k-1,k_m-1}^{MF}(q_j)$$

represents the probability that a married woman with realized state  $(k - 1, k_m - 1, q_j)$  will remain married and have an additional child. Now we can write the pre-determined part of the flow into the system as

$$g_{j+1}(k, k_m) = g_{k,k_m}^0(q_j) + \kappa_{k,k_m}^0(q_j) \sum_{q \in Q} f(q_j; q) M(k - 1, k_m - 1, q)$$

for any  $k_m \in \{0, 1, \dots, k\}$ , the law of motion is:

$$M(k, k_m, q_j) = \kappa_{k, k_m}^1(q_j) \sum_{q \in Q} f(q_j; q) M(k, k_m, q) + g_j(k, k_m)$$

, where

$$\kappa_{k, k_m}^1(q_j) \equiv (1 - \delta) [1 - \Phi(\varepsilon^*(k, k_m, q_j))] (1 - \pi_{k, k_m}^{MF}(q_j))$$

In terms of equation (17), the coefficients  $D_{1k, k_m} = [\kappa_{k, k_m}^1(q_j) f(q_j; q_i)]$ , and the vector of constants is  $D_{0k, k_m} = [g_1(k, k_m), \dots, g_{n_q}(k, k_m)]'$ .

#### A.4.3 case 3: $k > 0, k_m = 0$

For each  $k > 0$  with  $k_m = 0$ , we can also construct a linear system similar to that for  $k = 0$ , except with flows in from the population with  $k - 1$  kids.

**Singles** The result of the previous section means that the inflows to single status from married can be decomposed into a part with  $k_m = 0$  and a pre-determined part with  $k_m > 0$ .

For  $k > 0$ , the flows into  $N'_F(k)$  are from:

1. singles with  $k - 1$  children who didn't marry and then had a baby:

$$d_{11} \equiv (1 - \delta) \pi_{k-1}^{SF} (1 - \mu_{k-1}) N_F(k - 1)$$

2. singles with  $k$  children who didn't marry and didn't have a baby:

$$(1 - \delta) (1 - \pi_k^{SF}) (1 - \mu_k) N_F(k)$$

3. married with  $k - 1$  children who divorced and then had a baby:

$$d_{12} \equiv \pi_{k-1}^{SF} (1 - \delta) \sum_{k_m=0}^{k-1} \sum_{i=1}^{n_q} [\pi_{k-1, k_m}^D(q_i) M(k - 1, k_m, q_i)]$$

4. married with  $k$  children who divorced and did not have a baby:

$$(1 - \pi_k^{SF}) (1 - \delta) \sum_{i=1}^{n_q} [\pi_{k, 0}^D(q_i) M(k, 0, q_i) + d_{13}(q_i)]$$

, where

$$d_{13}(q_i) \equiv \sum_{k_m=1}^k \pi_{k, k_m}^D(q_i) M(k, k_m, q_i)$$

. The law of motion for single women is:

$$N'_F(k) = a_{11}N_F(k) + \sum_{i=1}^{n_q} a_{1i+1}M(k, 0, q_i) + d_1$$

where

$$\begin{aligned} a_{11} &= (1 - \delta) (1 - \pi_k^{SF}) (1 - \mu_k) \\ a_{1i+1} &= (1 - \pi_k^{SF}) (1 - \delta) \pi_{k,0}^D(q_i) \end{aligned}$$

and

$$d_1 = d_{11} + d_{12} + (1 - \delta) (1 - \pi_k^{SF}) \sum_{q \in Q} d_{13}(q)$$

**Married**,  $k_m = 0$  For married women in households with no kids from the husband, the flows into  $M'(k, 0, q_i)$  are:

1. From married with same number of wife's kids:

$$\kappa_k(q_i) \sum_{j=1}^{n_q} M(k, 0, q_j) f(q_i, q_j)$$

2. From single women with the same number of kids:

$$\kappa_k(q_i) f(q_i, \hat{q}) p_z (1 - \omega_0(k)) N_F(k)$$

, where

$$\kappa_k(q_i) = (1 - \delta) (1 - \pi_{k,0}^{MF}(q_i)) (1 - \Phi(\varepsilon^*(k, 0, q_i)))$$

. The full equations, written in terms of the linear system (16) are:

$$M'(k, 0, q_i) = \alpha_{i+1,1}N_F(k) + \alpha_{i+1,j+1}M(k, 0, q_j)$$

, where

$$\begin{aligned} \alpha_{i+1,1} &= \kappa_k(q_i) f(q_i, \hat{q}) p_z (1 - \omega_0(k)) \\ \alpha_{i+1,j+1} &= \kappa_k(q_i) f(q_i, q_j) \end{aligned}$$

. Therefore the coefficients  $C_{1k} = [a_{i,j}]$  and the constant terms are  $C_{0k} = [d_i]$ .

Statistic	Data		% Change	Source
	1970s	1990s		
Fraction married	0.74	0.53	-28%	Census 1970/2000
Fraction of kids in single mom households	0.13	0.22	69%	Living Arrangements of Children Under 18 Years Old: 1960 to Present
Fraction of mothers unmarried	0.19	0.25	32%	Census 1970/2000
Fraction of kids living with both Parents	0.82	0.62	-24%	Census 1970/2000
Share of fertility due to unmarried women	0.1	0.3	200%	NCHS Data Brief ■ No. 18 ■ May 2009
Fraction of families with mixed kids	0.06	0.1	67%	Census 1970/2000
Average birth rate	0.18	0.08	-56%	National Vital Statistics Reports, Vol. 48, No. 16, October 18, 2000

**Table 1:** *Changes in Aggregate Marital Indicators . Census computations based on women aged 18-44.*

Variable	1973		1995	
	NoKids	SinMom	NoKids	SinMom
Age	20.49 (2.210)	30.847 (3.151)	23.187 (5.411)	31.03 (4.462)
College Degree	0.11 (0.675)	0.054 (0.102)	0.312 (0.366)	0.138 (0.227)
Birth Rate	0.00039 (0.043)	0.006 (0.034)	0.002 (0.038)	0.005 (0.047)
Cohabiting	0.00011 (0.023)	0.00035 (0.008)	0.032 (0.139)	0.087 (0.185)
Attended College	0.474 (1.076)	0.12 (0.146)	0.533 (0.394)	0.244 (0.283)
High-School Diploma	0.938 (0.520)	0.563 (0.223)	0.843 (0.287)	0.739 (0.290)
Previously Married	0.006 (0.172)	0.216 (0.185)	0.067 (0.198)	0.557 (0.327)
Marriage Rate	0.00247 (0.107)	0.006 (0.036)	0.006 (0.061)	0.007 (0.055)

**Table 2:** *Descriptive Statistics.* NSFG 1973 and 1995 samples of single women aged 18-44. Sample for 1973 is reweighted to compensate absence of unmarried women without children.

Variable	1970-73		1990-95	
	birth	mar	birth	mar
Intercept	-5.603 (0.006)	-4.183 (0.004)	-4.012 (0.003)	-4.666 (0.002)
1 Kid	0.758 (0.001)	-0.407 (0.001)	0.226 (0.001)	0.048 (0.001)
2 Kids	-0.031 (0.001)	-0.094 (0.001)	-0.060 (0.000)	-0.018 (0.000)
3 Kids	0.061 (0.001)	0.151 (0.001)	-0.037 (0.001)	-0.051 (0.001)
4 Kids	-0.016 (0.002)	-0.225 (0.002)	0.024 (0.001)	-0.123 (0.001)
5 Kids	0.377 (0.002)	0.201 (0.002)	0.115 (0.002)	0.139 (0.002)
Cohabiting	1.073 (0.005)	-2.957 (0.586)	-0.058 (0.001)	0.054 (0.000)
Previously Married	0.051 (0.001)	0.671 (0.001)	0.194 (0.000)	-1.588 (0.004)
Age	0.194 (0.000)	0.081 (0.000)	0.026 (0.001)	-0.088 (0.000)
Age Squared	-0.004 (0.000)	-0.002 (0.000)	0.142 (0.000)	0.169 (0.000)
Age x BA	0.014 (0.000)	-0.010 (0.000)	-0.003 (0.000)	-0.003 (0.000)
Attending School	-0.107 (0.003)	-0.112 (0.001)	-0.325 (0.001)	-0.282 (0.000)
HS diploma	0.442 (0.001)	1.371 (0.000)	-0.177 (0.000)	0.042 (0.000)
College	-0.184 (0.001)	0.030 (0.001)	-0.200 (0.001)	-0.040 (0.000)
College Degree	-0.394 (0.009)	0.273 (0.003)	-0.268 (0.001)	0.008 (0.000)

**Table 3:** *Probit Estimates of unmarried women's monthly marriage and birth rates. Samples from the NSFG, 1973 and 1995 waves.*

Value	Parameter
<b>Normalized Parameters</b>	
1.00	standard deviation of persistent quality shock
1.00	probability that marriages are allowed to occur
2.00	divorce cost
1.00	standard deviation of transitory quality shock
1.00	male entry cost into singles market
2.00	utility flow of child-less marriages
<b>Fixed Parameters</b>	
0.96	Discount factor
0.05	Exit rate into inactive status (sterility)
0.30	Maximum rate of Married Fertility
0.22	Maximum rate of unmarried fertility
0.10	Effort-fertility curvature parameter

**Table 4(a):** *Normalized and Fixed Parameter Values in Benchmark calibration.*

Calibration		Parameter
1973	1995	
-0.05	-0.008	Effect of Step-Kids on Marriage Output
0.02		Womens utility for kids, intercept
-0.01		Mens utility for kids, intercept
-0.035		Women's utility for kids, slope
0.005		Men's utility for kids, slope
0.524		Persistence of match quality
0.394		Fertility-choice cost slope
-0.511	-0.487	Utility bonus for singles
2	2.226	Divorce Cost

**Table 4(b):** *Free-Parameter Values for Calibrated Models.* Parameter values for 1995 are fixed at the 1975 values, except for those listed in the 1995 column.



1973		1995		Statistic
NSFG	Model	NSFG	Model	
0.23	0.249	0.112	0.135	Marriage Rate, non-mothers
0.13	0.111	0.126	0.117	Marriage rate, mothers of one child
0.091	0.086	0.118	0.118	Marriage rate, mothers of two children
0.227	0.218	0.224	0.228	Birth Rate, married couples with no kids
0.187	0.188	0.215	0.21	Birth Rate, married couples with one kid
0.098	0.107	0.1	0.1	Birth Rate, married couples with two kids
0.019	0.017	0.046	0.045	Birth rate, single women with no kids
0.056	0.065	0.09	0.053	Birth rate, single women with one child
0.02	0.021	0.049	0.043	Divorce Rate

**Table 5(a): Calibration of Marriage and Fertility in Benchmark Model.** NSFG statistics consist of Age-25 predictions from estimated age profiles, controlling for education and cohabitation etc. Unshaded cells correspond to calibration targets.

1973		1995		Statistic
Data	Model	Data	Model	
0.74	0.74	0.53	0.48	Fraction married
0.13	0.06	0.22	0.28	Fraction of kids in single-mom households
0.82	0.92	0.62	0.65	Fraction of kids living with both parents
0.10	0.06	0.30	0.26	Share of fertility due to unmarried women
0.06	0.02	0.10	0.08	Fraction of families with mixed kids
0.18	0.15	0.08	0.13	Average birth rate

**Table 5(b): Results from NSFG Surveys and Benchmark Model Calibrations.** Based on population of women aged 18-44.

		Kids to Date (Parity)		
Single		0	1	2
		0.035	0.082	0.163
Married		Husband's Kids		
q	Wife's Kids	0	1	2
-1.62	0	0.30	--	--
	1	0.30	0.21	--
	2	0.19	0.16	0.12
0	0	0.30	--	--
	1	0.30	0.20	--
	2	0.19	0.16	0.12
1.62	0	0.30	--	--
	1	0.30	0.19	--
	2	0.19	0.15	0.12

**Table 6(a):** Fertility rates of active women in benchmark model.

Kids	Marriage Probability	Men/Women	Surplus	Probability Wife Gets Surplus	Y_mar	VE_SF
0	0.39	0.53	1.79	0.10	22.04	10.70
1	0.15	0.18	1.31	0.01	20.04	9.03
2	0.08	0.10	1.21	0.00	19.35	8.44
3	0.00	0.00	--	0.00	--	8.20

**Table 6(b):** Marriage rates of active women in benchmark model. Excess supply of single males = 0.092.

Age	Non-Moms				Single Moms			
	1973		1995		1973		1995	
	Marr-73	Birth-73	Marr-95	Birth-95	Marr-73	Birth-73	Marr-95	Birth-95
21	0.301	0.021	0.117	0.072	0.122	0.169	0.103	0.134
22	0.304	0.022	0.122	0.069	0.124	0.175	0.108	0.128
23	0.305	0.022	0.126	0.065	0.125	0.178	0.111	0.122
24	0.304	0.022	0.128	0.061	0.124	0.177	0.113	0.114
25	0.301	0.021	0.130	0.056	0.122	0.173	0.114	0.106
26	0.296	0.020	0.130	0.051	0.120	0.166	0.115	0.098
27	0.289	0.019	0.129	0.046	0.116	0.156	0.114	0.090
28	0.280	0.017	0.127	0.041	0.111	0.143	0.112	0.081
29	0.269	0.014	0.123	0.037	0.106	0.129	0.109	0.073
30	0.257	0.012	0.119	0.032	0.100	0.113	0.105	0.064
31	0.243	0.010	0.114	0.028	0.093	0.097	0.100	0.056
32	0.229	0.008	0.108	0.024	0.086	0.081	0.095	0.049
33	0.213	0.006	0.101	0.020	0.079	0.066	0.088	0.042
34	0.196	0.004	0.093	0.017	0.071	0.052	0.082	0.035
35	0.179	0.003	0.086	0.014	0.064	0.040	0.075	0.029
36	0.162	0.002	0.078	0.011	0.056	0.029	0.068	0.024
37	0.145	0.001	0.070	0.009	0.049	0.021	0.061	0.020
38	0.128	0.001	0.062	0.007	0.042	0.014	0.053	0.016
39	0.111	0.001	0.054	0.005	0.036	0.009	0.047	0.012
40	0.096	0.000	0.046	0.004	0.030	0.006	0.040	0.009
41	0.082	0.000	0.039	0.003	0.025	0.003	0.034	0.007
42	0.068	0.000	0.033	0.002	0.020	0.002	0.028	0.005
43	0.056	0.000	0.027	0.002	0.016	0.001	0.023	0.004
44	0.046	0.000	0.022	0.001	0.013	0.001	0.019	0.000

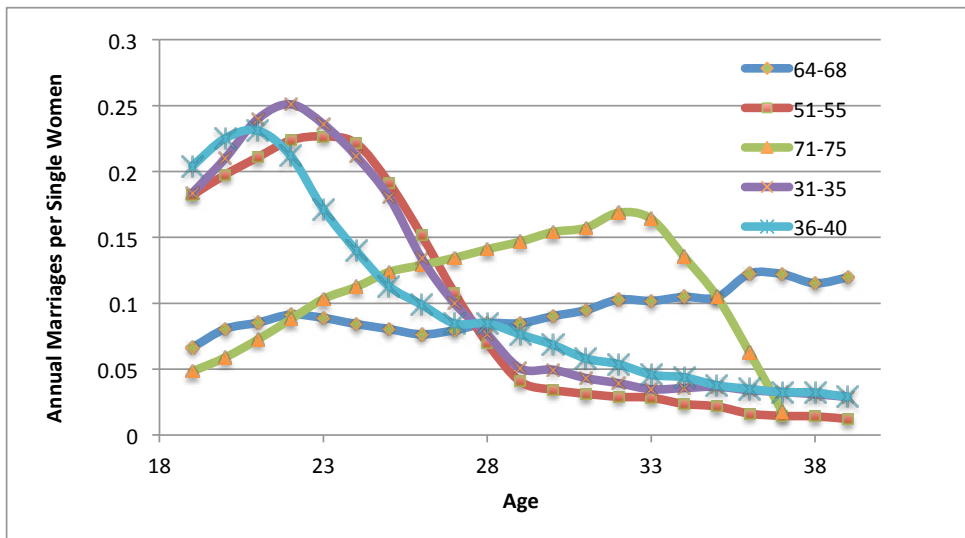
**Table A1:** *Predicted Age Profiles for Marriage and Child-Birth.* Based on probit regression estimates on single-woman samples from NSFG 1973 and 1995. To compensate lack of never-married non-mothers in sample design, 1973 sample re-weighted using 1970 census.

Variable	1973	1995
Intercept	-2.9897 (0.003)	-3.1434 (0.003)
1 Kid	-0.0048 (0.000)	0.0598 (0.000)
2 Kids	-0.1004 (0.000)	-0.0186 (0.000)
3 Kids	-0.2567 (0.000)	-0.3201 (0.000)
4 Kids	-0.0160 (0.000)	-0.0552 (0.000)
5 Kids	0.0793 (0.001)	0.0276 (0.001)
Cohabiting	0.1619 (0.001)	0.0036 (0.002)
Previously Married	0.0951 (0.001)	-0.0247 (0.002)
Age	0.1054 (0.000)	0.1190 (0.000)
Age Squared	-0.0025 (0.000)	-0.0028 (0.000)
Age x BA	0.0237 (0.000)	0.0327 (0.000)
Attending School	-0.2405 (0.002)	-0.1408 (0.000)
HS diploma	-0.0513 (0.000)	-0.0789 (0.000)
College	-0.0719 (0.000)	0.0123 (0.000)
College Degree	-0.6788 (0.003)	-0.9559 (0.001)

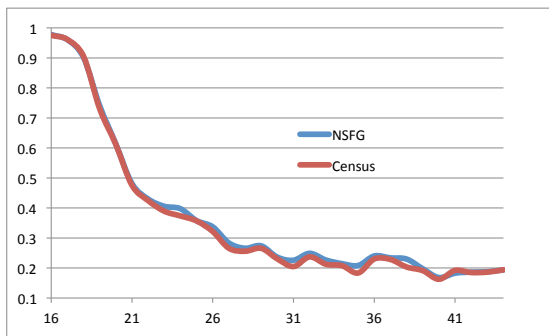
**Table A2:** Probit Estimates of married women's monthly birth rates. Samples from the NSFG, 1973 and 1995 waves.

Age	Non-Moms		Moms	
	NSFG	Model	NSFG	Model
21	0.2818	0.2761	0.2188	0.2309
22	0.269	0.2741	0.2094	0.2291
23	0.2533	0.2695	0.1999	0.225
24	0.2331	0.2622	0.1903	0.2185
25	0.2112	0.2525	0.1806	0.21
26	0.1921	0.2405	0.1709	0.1994
27	0.1747	0.2265	0.1611	0.1872
28	0.1602	0.2108	0.1511	0.1735
29	0.1458	0.1937	0.1399	0.1587
30	0.1305	0.1757	0.1284	0.1432
31	0.1158	0.1571	0.1168	0.1273
32	0.101	0.1383	0.1051	0.1115
33	0.087	0.1199	0.0936	0.096
34	0.0735	0.1022	0.0824	0.0813
35	0.0616	0.0855	0.0718	0.0676
36	0.044	0.0702	0.0628	0.0551
37	0.0299	0.0565	0.0541	0.044
38	0.0198	0.0445	0.0454	0.0344
39	0.0139	0.0343	0.0368	0.0263
40	0.0079	0.0258	0.0283	0.0196
41	0.0056	0.019	0.02	0.0143
42	0.0048	0.0136	0.0117	0.0101
43	0.0038	0.0094	0.0036	0.007
44	0.0021	0.0064	-0.0044	0.0047

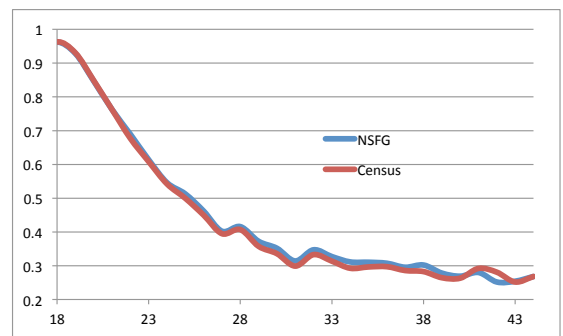
**Table A3:** Predicted Age-Birthrate profiles, based on Probit Estimates of married women's monthly birth rates.



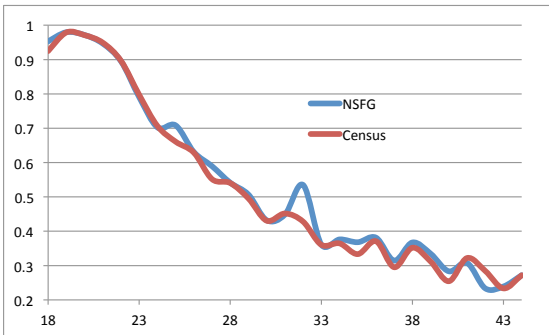
**Figure 1:** *Marriage Rates by Birth Cohort.* Based on US Census, 1980, 2000 and 2010 waves.



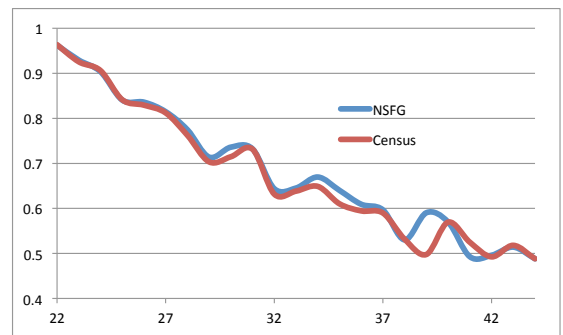
**Figure 2(a):** Women never-married, never given birth; dropouts. Census 1970 versus reweighted version of NSFG samples.



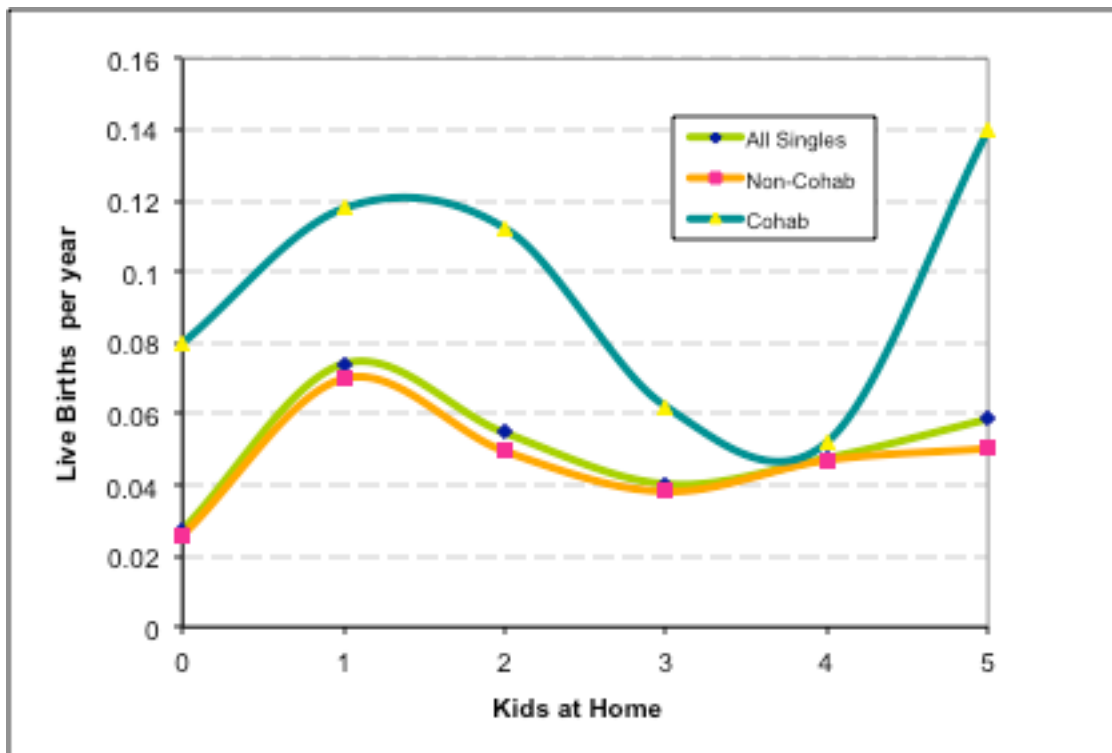
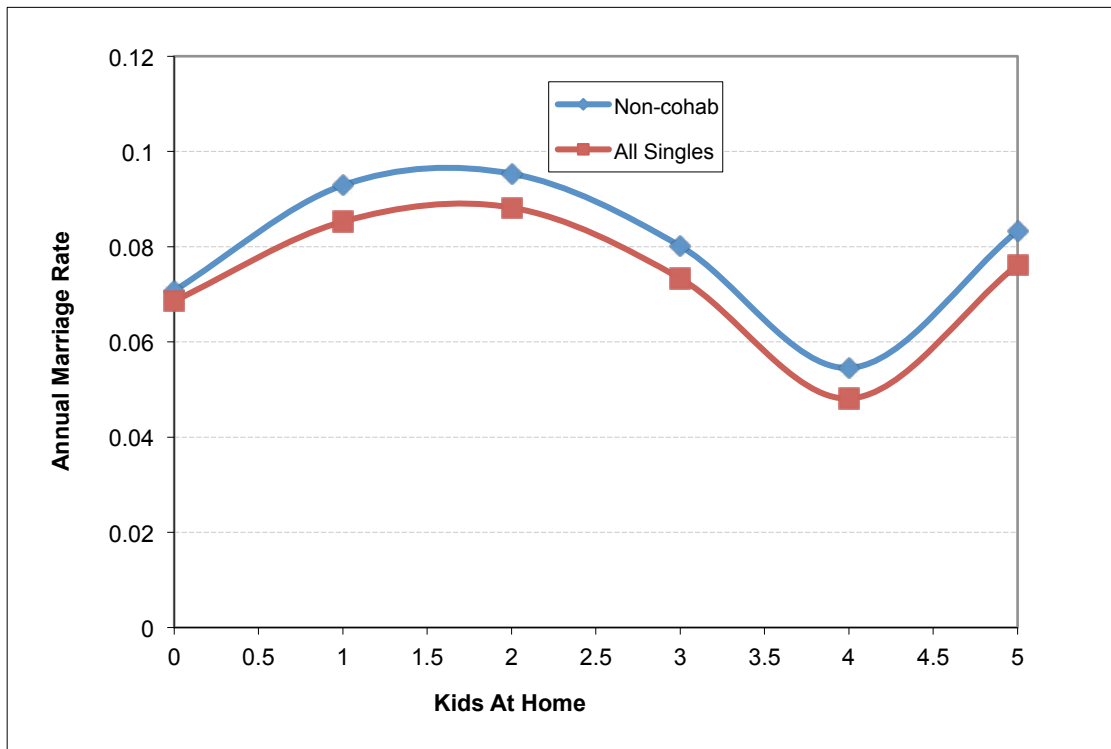
**Figure 2(b):** Women never-married, never given birth; High-school graduates. Census 1970 versus reweighted version of NSFG samples.



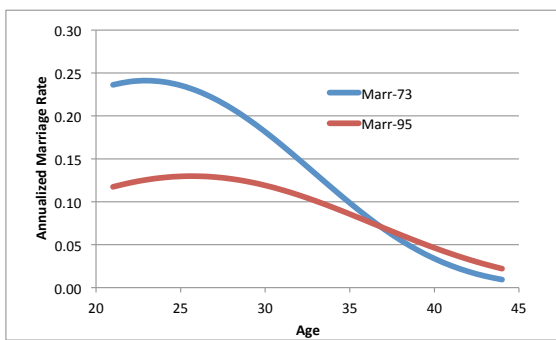
**Figure 2(c):** Women never-married, never given birth; attended college. Census 1970 versus reweighted version of NSFG samples.



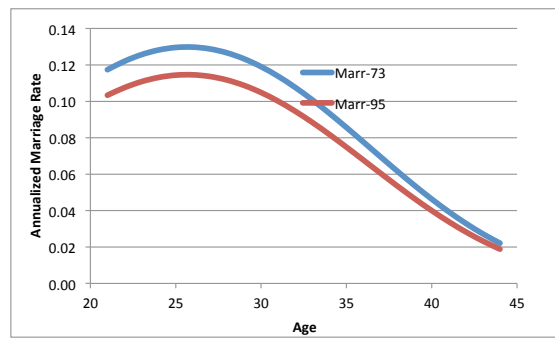
**Figure 2(b):** Women never-married, never given birth; College graduates. Census 1970 versus reweighted version of NSFG samples.



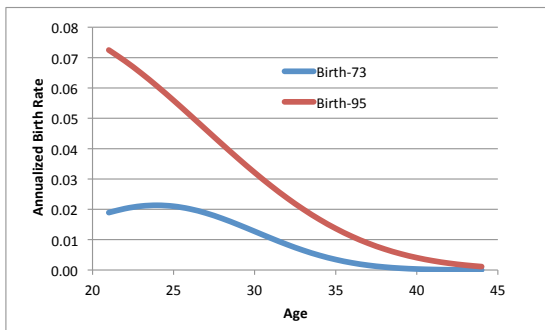
**Figure 3:** Single and Cohabitants in the 1995 NSFG. Annualised marriage and birth rates per single woman.



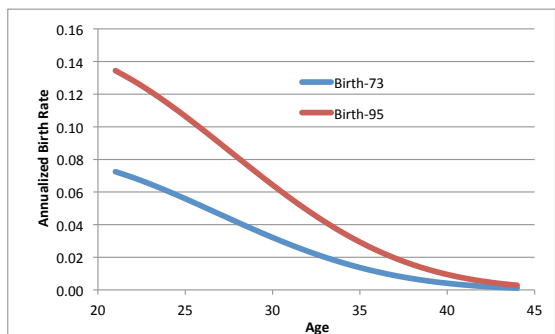
**Figure 4(a):** Estimated Age Profiles for Marriage Rates; Single Women with no children.



**Figure 4(b):** Estimated Age Profiles for Marriage Rates; Single Women with one child.



**Figure 4(c):** Estimated Age Profiles for Birth Rates; Single women without children.



**Figure 4(d):** Estimated Age Profiles for Birth Rates; Single Women with one child.



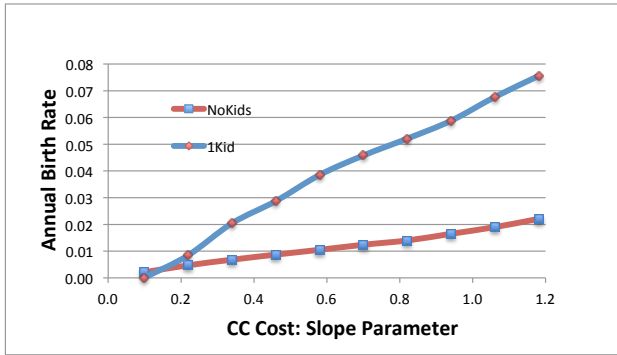


Figure 5(a): Effect on unmarried birthrates of changing birth-control costs; benchmark value =0.39.

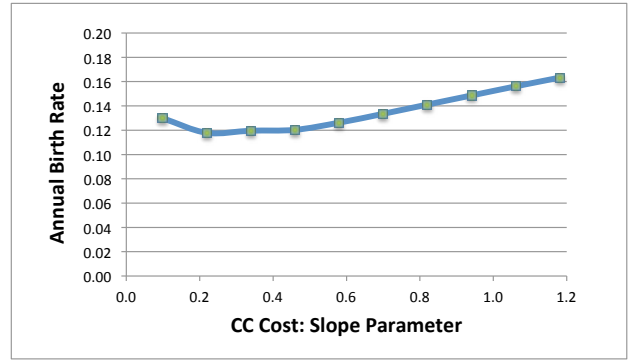


Figure 5(b): Effect on aggregate birthrates of changing birth-control costs; benchmark value =0.39.

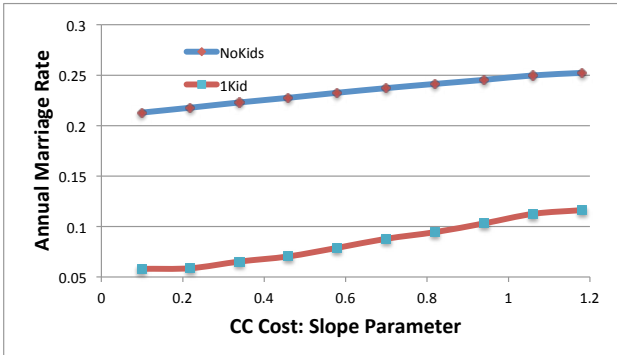


Figure 5(c): Effect on marriage rates of changing birth-control costs; benchmark value =0.39.

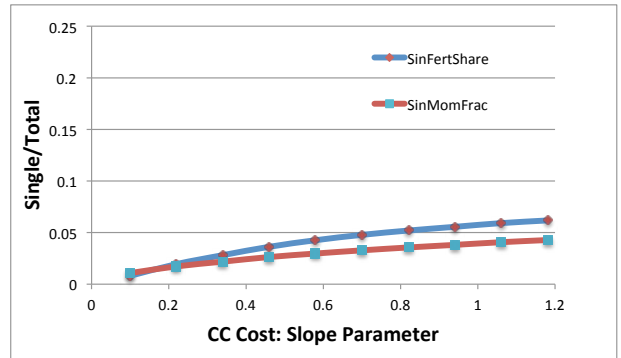


Figure 5(d): Effect on unmarried shares of birthrates and mothers of changing birth-control costs; benchmark value =0.39.

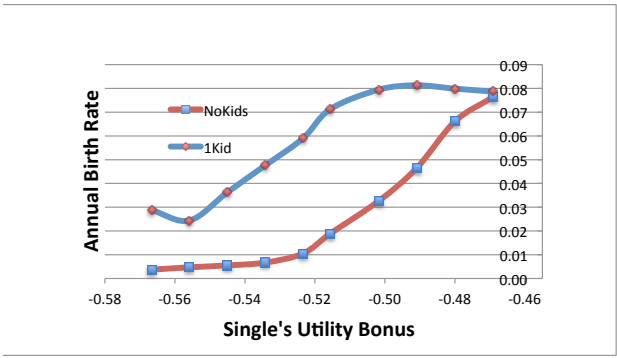


Figure 6(a): Effect on unmarried birthrates of changing unmarried utility; benchmark value =0.55.

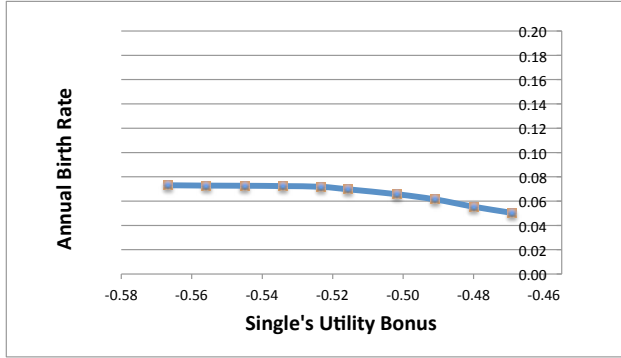


Figure 6(b): Effect on aggregate birthrates of changing unmarried utility; benchmark value =0.55

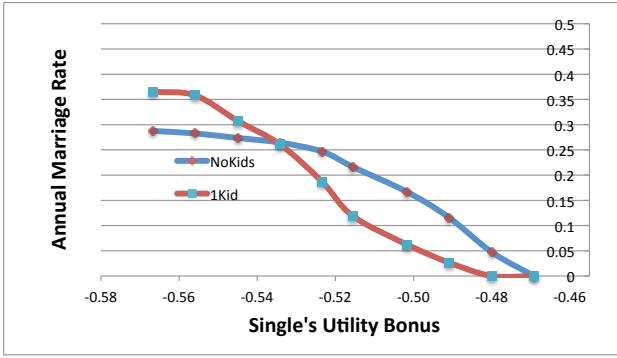


Figure 6(c): Effect on marriage rates of changing unmarried utility; benchmark value =0.55.

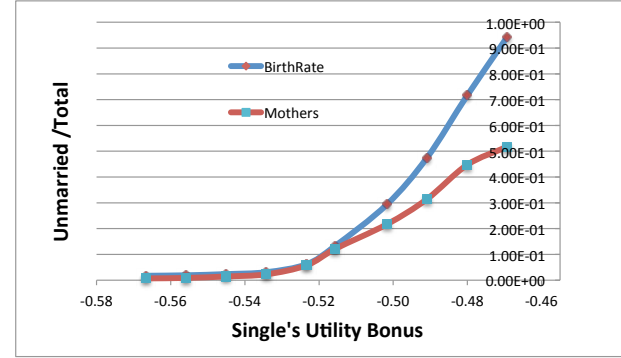


Figure 6(d): Effect on marriage rates of changing unmarried utility; benchmark value =0.55

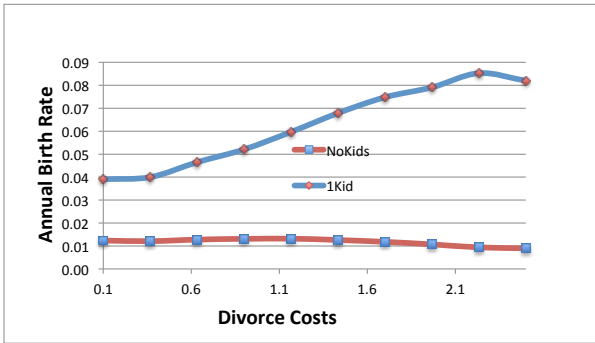


Figure 7(a): Effect on unmarried birthrates of changing divorce costs; benchmark value =2.0.

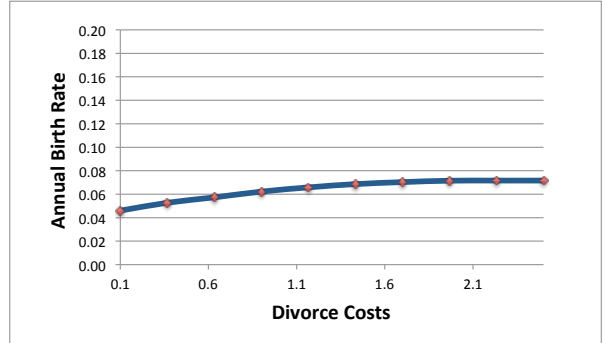


Figure 7(b): Effect on aggregate birthrates of changing divorce costs; benchmark value =2.0.

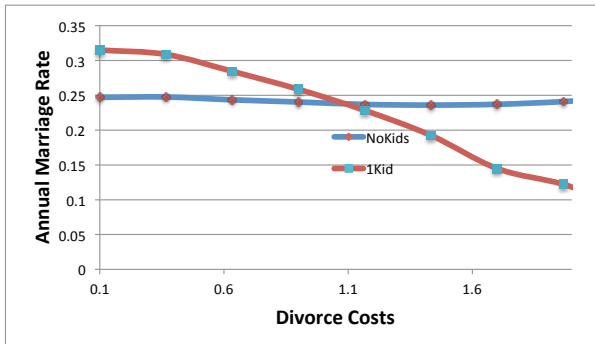


Figure 7(c): Effect on marriage rates of changing divorce costs; benchmark value =2.0.

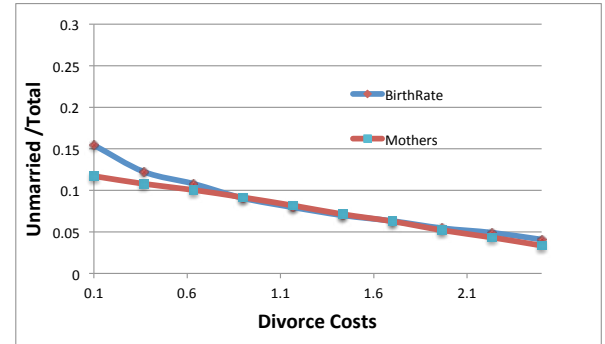


Figure 7(d): Effect on unmarried shares of birthrates and of mothers of changing divorce costs; benchmark value =2.0.

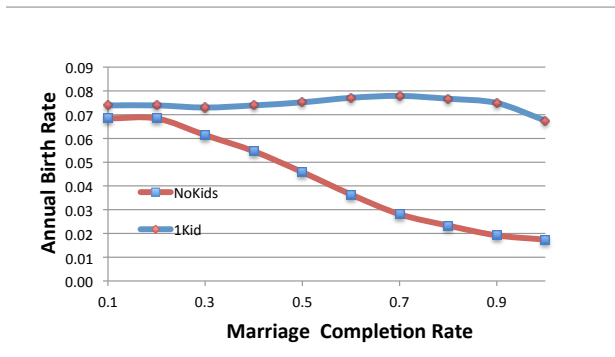


Figure 8(a): Effect on unmarried birthrates of changing marriage friction; benchmark value =2.0.

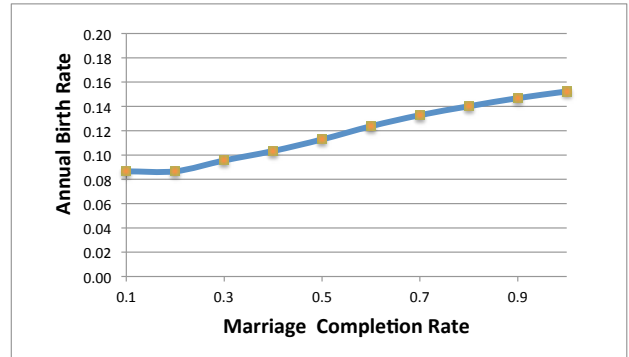


Figure 8(b): Effect on aggregate birthrates of changing marriage friction; benchmark value =2.0.

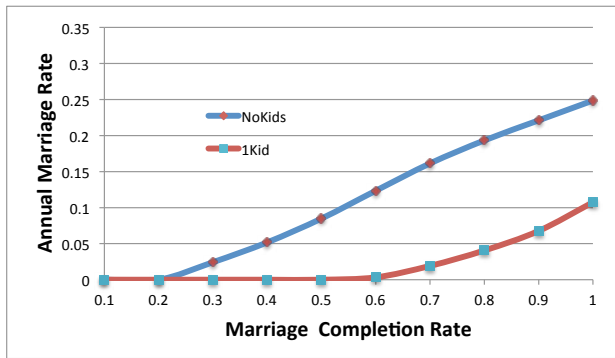


Figure 8(c): Effect on marriage rates of changing marriage friction; benchmark value =2.0.

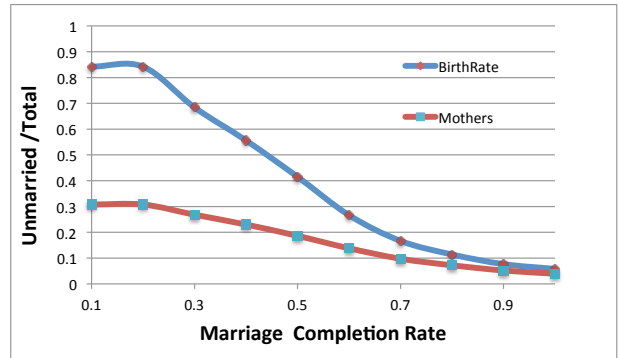
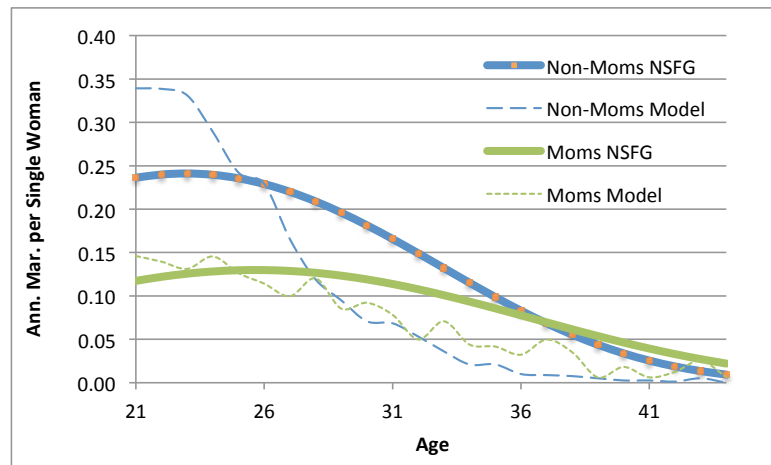
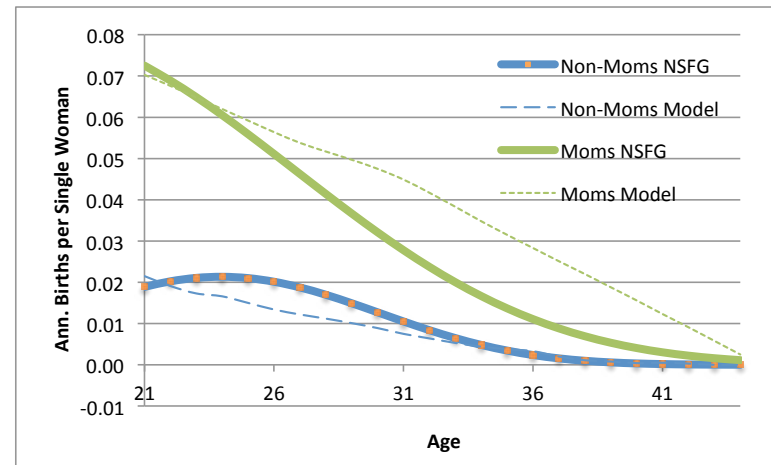


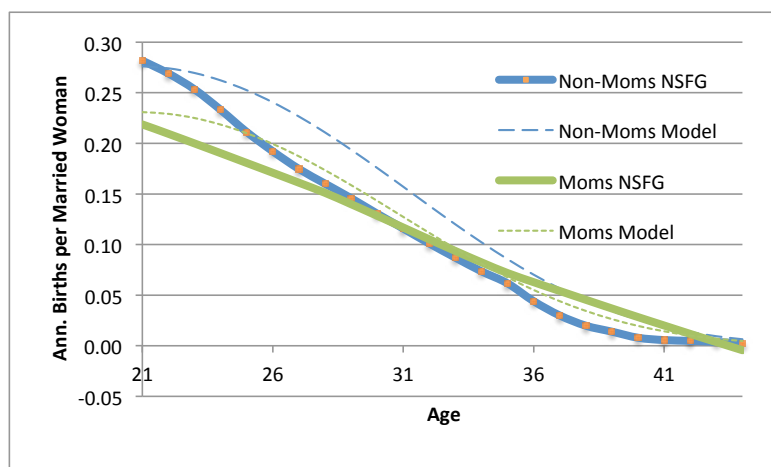
Figure 8(d): Effect on unmarried shares of births and mothers of changing marriage friction; benchmark value =2.0.



**Figure 9(a):** Marriage Rates in the benchmark model. NSFG series based on Probit predictions from 1973 wave; model series based on 1973 benchmark simulation.



**Figure 9(b):** Unmarried fertility rates in the benchmark model. NSFG series based on Probit predictions from 1973 wave; model series based on 1973 benchmark simulation.



**Figure 9(c):** Married fertility rates in the benchmark model. NSFG series based on Probit predictions from 1973 wave; model series based on 1973 benchmark simulation.

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